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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK II

BY

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PREFACE

THIS series of mathematical textbooks marks a new type of mathematics as to aims, purposes, and material. Unbiased by tradition, the author seeks to give the mathematics necessary in order to interpret the quantitative phases of life, met by the average intelligent person outside of his specialized vocation.

The material, selected with this aim in view, is naturally based upon some social issue. Such a selection of material not only leads to the habit of using the mathematics learned in school, to interpret the quantitative side of everyday life that is met out of school, but it will also greatly increase the interest in the subject — a vital factor in the economy of learning.

It should be noticed, then, that this series especially emphasizes the interpretative function of mathematics, and seeks to develop both the *power* to see and the *habit* of seeing the quantitative relationships that necessarily arise in topics of general conversation and reading.

To do this, the series makes use of concepts and processes usually classed as arithmetic, algebra, geometry, and trigonometry; but it uses only such a part of these subjects as is needed to interpret references met in general reading. It is the needs of the student, then, that is kept constantly in mind in the selection of material, and not the development of the subject or traditional subject matter.

Book II reviews methods of computation and introduces a few of the most-used "short cuts." The formula is reviewed and its use extended, in order that the student may be able to interpret its meaning and to evaluate it when met in other work, and that he may see its advantages and use, in expressing quantitative relationships in the briefest possible forms.

The simple equation of one unknown quantity is introduced to acquaint the student with its meaning and with methods of solving it, and as a necessary tool in solving problems of proportion that follow. Ratio and proportion precede a study of similar figures, to furnish a means of expressing relations found through measurement, and to furnish a tool to use when applying the properties of similar figures to the finding of heights and distances.

The study of similar figures naturally leads to scale drawing and trigonometric ratios as means of finding heights and distances without actually measuring them. In **Book II**, only the tangent relation is used.

A very complete discussion of the graph as used in representing quantitative relations is given. Instead of graphs being made for the occasion, they are taken from leading newspapers, magazines, and other sources, thus leading the student to observe and interpret them as he meets them in general reading. The functional graph is discussed briefly.

After a thorough discussion of relations expressed by per cent, the last half of **Book II** discusses business terms, forms, and processes; banking; methods of investing money; the meaning and nature of insurance; and the meaning and necessity of taxes. The problems under these topics are real and are given for the purpose of helping the student to interpret these important topics which are of interest to all of us. The unreal, indirect problems, still found in most courses — given through adherence to tradition or for mental gymnastics — are carefully eliminated as contributing in no way to the aims of this series.

Properly used, then, this series of textbooks will give an interpretative power to mathematics not developed by the methods and the problem material of the present type of textbooks written for these grades.

JOHN C. STONE.

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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK II

CHAPTER I

REVIEW OF ARITHMETICAL PROCESSES: SHORT METHODS

You have learned how to compute with whole numbers, fractions, and decimals, but continued practice is necessary, in order to develop greater skill; that is, in order to be more accurate and rapid. Skill in computation, however, depends not only upon recalling the number facts accurately and rapidly, but upon seeing relations that will save figures and even whole processes. In this chapter, then, will be discussed some "short cuts" in computation. These may occur chiefly in multiplication and division.

1. ADDITION: WHOLE NUMBERS, FRACTIONS, AND DECIMALS

In all computation, one should form the habit of going over the work a second time to see if it is correct. This is called **checking the work.**

In addition, check by adding a second time in reverse order.

One method of recording the steps in addition and checking the work is shown in the following example.

WORK	CHECK
495	41
628	43
786	40
549	
875	
698	
4031	

EXPLANATION. — The sum of the first column is 41. It is written under "check." The sum of the next, with the 4 carried, is 43. The sum of the next, with 4 carried, is 40.

Now, beginning with the highest order and adding in the opposite direction, the sum is 36; this with the 4 of the next lower sum (43) is 40. The next sum is 39, which with 4 (from 41) is 43. And the next is 41. So the sum 4031 is recorded. Some prefer the check shown in **Book I**. This is just a little shorter. Use the method you prefer.

Drill Exercises

Add and check:

1.	2.	3.	4.	5.
79,648	65,468	56,981	67,943	68,991
84,796	96,394	98,156	59,946	54,036
72,387	54,387	86,675	78,897	73,938
86,793	86,731	17,386	27,389	42,392
54,635	72,467	39,756	82,137	57,169
68,357	81,120	79,346	64,938	42,627
53,386	27,351	42,932	58,716	63,128
49,962	84,936	93,742	78,345	96,321
51,195	35,426	64,396	27,564	38,528
16,840	26,734	46,321	54,375	16,784
31,178	52,191	32,875	41,728	36,597
28,385	16,207	42,368	50,732	48,096

Adding Two Numbers Without a Pencil

In adding two numbers of two figures each, without a pencil, it is easiest to add the *tens* to one of the numbers and to that sum add the *ones*. Thus, in adding 48 and 56, think 98, 104; or in adding 27 and 58, think 77, 85.

Drill Exercises

Add at sight:

1. 42 + 56.	11. 19 + 35.	21. 87 + 56.
2. 27 + 35.	12. 46 + 18.	22. 95 + 48.
3. 38 + 53.	13. 55 + 28.	23. 86 + 93.
4. 41 + 29.	14. 31 + 67.	24. 78 + 84.
5. 36 + 29.	15. 52 + 27.	25. 57 + 69.
6. 27 + 56.	16. 48 + 39.	26. 48 + 88.
7. 54 + 17.	17. 63 + 29.	27. 54 + 76.
8. 38 + 46.	18. 57 + 17.	28. 85 + 47.
9. 35 + 49.	19. 82 + 49.	29. 96 + 58.
10. 73 + 19.	20. 98 + 54.	30. 87 + 98.

These exercises are to fix the *method*. Before this, or any other form of computation, is of value, it must become a *habit*. So, by using this method whenever the sum of any two numbers of two figures each is wanted, you will soon find it easy to add all such numbers without a pencil.

Adding a Number Nearly 100, 1000, etc.

To add 98 to 56, observe that it is 2 less than 100 to add, hence, the sum is $100 + 56 - 2$ or $156 - 2$ or 154.

$996 + 748 = 1744$, for it is $1748 - 4$. Why?

Drill Exercises

At sight add:

1. 998 + 645.	6. 875 + 970.	11. 995 + 846.
2. 990 + 875.	7. 694 + 920.	12. 970 + 645.
3. 995 + 767.	8. 736 + 998.	13. 980 + 763.
4. 990 + 843.	9. 645 + 980.	14. 991 + 750.
5. 996 + 735.	10. 563 + 940.	15. 997 + 645.

Drill Exercises

Add and check:

1.	2.	3.	4.	5.
38.4	16.48	3.96	74.5	63.48
75.64	24.3	98.6	9.16	8.34
9.15	8.57	6.98	18.45	16.9
16.9	9.875	19.8	35.64	9.85
7.63	10.48	13.65	17.29	48.76
9.375	6.847	9.268	7.175	9.43
42.86	75.37	54.28	16.97	16.38
4.463	7.289	7.543	6.297	5.783
19.91	14.34	16.28	42.98	53.9

6. In adding decimals, why are the decimal points written under each other?

Drill Exercises

Add and check:

1. $\frac{3}{4} + \frac{1}{8} + \frac{1}{2}$.
2. $\frac{3}{5} + \frac{7}{10} + \frac{3}{4}$.
3. $\frac{7}{8} + \frac{1}{2} + \frac{5}{16}$.
4. $\frac{2}{5} + \frac{7}{15} + \frac{19}{30}$.
5. $\frac{2}{3} + \frac{5}{6} + \frac{7}{12}$.
6. $\frac{1}{3} + \frac{5}{6} + \frac{13}{18}$.

7. In adding fractions, why are they first changed to a common denominator?

8. What are fractions called whose numerators are equal to or greater than the denominators?

9. How are improper fractions changed to whole or mixed numbers?

Add and check:

10.	11.	12.	13.	14.
$9\frac{1}{2}$	$26\frac{1}{3}$	$24\frac{1}{2}$	$32\frac{1}{3}$	$28\frac{2}{5}$
$8\frac{3}{4}$	$48\frac{1}{6}$	$16\frac{3}{4}$	$46\frac{5}{6}$	$42\frac{1}{3}$
$7\frac{5}{8}$	$54\frac{2}{3}$	$42\frac{7}{16}$	$16\frac{13}{18}$	$26\frac{2}{3}$
$6\frac{1}{2}$	$38\frac{1}{2}$	$36\frac{1}{8}$	$9\frac{2}{3}$	$41\frac{1}{5}$
$8\frac{1}{8}$	$19\frac{3}{4}$	$24\frac{1}{4}$	$7\frac{2}{9}$	$36\frac{2}{15}$

Adding Special Fractions

In all computation, one should be on the alert for combinations that will save work. Thus, to add $\frac{1}{4} + \frac{5}{8} + \frac{3}{4} + \frac{3}{8} + \frac{1}{2}$, one should observe that $\frac{1}{4} + \frac{3}{4} = 1$, and that $\frac{5}{8} + \frac{3}{8} = 1$. Hence, the sum is $2\frac{1}{2}$. And to add $\frac{1}{3} + \frac{1}{5}$, one should observe that since 3 and 5 have no common factors and since each numerator is 1, the *new* numerators will be 5 and 3, respectively, and the denominator is 3×5. Hence, the sum is $\frac{8}{15}$. That is, the sum is the sum of the two denominators over their product.

Drill Exercises

Give at sight:

1. $\frac{1}{3} + \frac{1}{4}$.
2. $\frac{1}{2} + \frac{1}{3}$.
3. $\frac{1}{3} + \frac{1}{7}$.
4. $\frac{1}{4} + \frac{1}{5}$.
5. $\frac{1}{2} + \frac{1}{5}$.
6. $\frac{1}{3} + \frac{1}{8}$.
7. $\frac{1}{4} + \frac{1}{7}$.
8. $\frac{1}{2} + \frac{1}{9}$.
9. $\frac{1}{3} + \frac{1}{10}$.
10. $\frac{1}{6} + \frac{1}{5}$.
11. $\frac{1}{6} + \frac{1}{7}$.
12. $\frac{1}{8} + \frac{1}{9}$.
13. $\frac{1}{5} + \frac{1}{7}$.
14. $\frac{1}{8} + \frac{1}{11}$.
15. $\frac{1}{10} + \frac{1}{11}$.
16. $\frac{1}{6} + \frac{1}{11}$.
17. $\frac{2}{3} + \frac{3}{4} + \frac{1}{3} + \frac{1}{4} + \frac{3}{8}$.
18. $\frac{1}{4} + \frac{3}{8} + \frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.
19. $\frac{1}{2} + \frac{3}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{2}$.
20. $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{2} + \frac{5}{6}$.
21. $\frac{1}{7} + \frac{1}{2} + \frac{3}{4} + \frac{6}{7} + \frac{1}{4}$.
22. $\frac{1}{9} + \frac{1}{2} + \frac{1}{9} + \frac{1}{2} + \frac{7}{9}$.
23. $\frac{3}{4} + \frac{1}{8} + \frac{1}{2} + \frac{7}{8} + \frac{1}{2}$.
24. $\frac{3}{5} + \frac{7}{8} + \frac{5}{6} + \frac{2}{5} + \frac{1}{8}$.

2. SUBTRACTION: WHOLE NUMBERS, FRACTIONS, AND DECIMALS

Subtraction is the inverse of addition. That is, the sum of two addends, and one of the two addends, are given, and the other addend is to be found. The given sum is called the **minuend**. The given addend is called the **subtrahend**. The addend found is called the **remainder** or **difference**.

Always check subtraction by adding the result to the subtrahend to see if it equals the minuend.

Do not rewrite, but check the result as it stands.

Drill Exercises

Subtract and check:

1.	**2.**	**3.**	**4.**	**5.**
65,179	72,307	61,302	29,361	30,621
28,396	19,698	28,496	17,975	13,794
6.	**7.**	**8.**	**9.**	**10.**
81,106	71,193	40,069	61,110	52,903
47,647	53,465	28,773	28,326	49,628
11.	**12.**	**13.**	**14.**	**15.**
64,216	81,726	42,901	61,110	53,306
28,269	54,392	16,284	49,306	48,729
16.	**17.**	**18.**	**19.**	**20.**
35.48	204.3	39.026	42.16	7.36
16.527	165.48	16.48	19.865	5.475
21.	**22.**	**23.**	**24.**	**25.**
38.465	42.065	13.065	47.081	52.375
19.8	19.78	9.87	19.765	48.9

26. In the subtraction of decimals, why are the decimal points placed under each other?

27. In the subtraction of fractions, why must the fractions be changed to common denominators?

Subtract and check:

28.	**29.**	**30.**	**31.**	**32.**
$35\frac{4}{5}$	$42\frac{1}{5}$	$30\frac{1}{3}$	$24\frac{1}{2}$	$48\frac{1}{2}$
$15\frac{1}{5}$	$15\frac{4}{5}$	$16\frac{2}{3}$	$16\frac{1}{4}$	$17\frac{3}{4}$
33.	**34.**	**35.**	**36.**	**37.**
$260\frac{2}{3}$	$204\frac{1}{2}$	$240\frac{5}{6}$	$305\frac{3}{8}$	$201\frac{3}{4}$
$192\frac{5}{6}$	$146\frac{2}{3}$	$198\frac{2}{3}$	$168\frac{1}{2}$	$128\frac{7}{8}$

Subtracting Special Fractions

When the numerators are each 1 and the denominators have no common factor, the fractions may be subtracted at sight. Thus, $\frac{1}{4} - \frac{1}{7} = \frac{3}{28}$, for it is seen that the *new* numerators will be 7 and 4 respectively and the common denominator will be 4×7, or 28. That is, the result is the difference of the numerators over their product.

Subtract at sight:

1. $\frac{1}{2} - \frac{1}{5}$.	**6.** $\frac{1}{3} - \frac{1}{7}$.	**11.** $\frac{1}{3} - \frac{1}{10}$.	**16.** $\frac{1}{2} - \frac{1}{11}$.
2. $\frac{1}{4} - \frac{1}{5}$.	**7.** $\frac{1}{3} - \frac{1}{8}$.	**12.** $\frac{1}{4} - \frac{1}{9}$.	**17.** $\frac{1}{3} - \frac{1}{11}$.
3. $\frac{1}{3} - \frac{1}{5}$.	**8.** $\frac{1}{4} - \frac{1}{7}$.	**13.** $\frac{1}{5} - \frac{1}{7}$.	**18.** $\frac{1}{4} - \frac{1}{11}$.
4. $\frac{1}{2} - \frac{1}{3}$.	**9.** $\frac{1}{5} - \frac{1}{6}$.	**14.** $\frac{1}{6} - \frac{1}{7}$.	**19.** $\frac{1}{4} - \frac{1}{15}$.
5. $\frac{1}{2} - \frac{1}{7}$.	**10.** $\frac{1}{3} - \frac{1}{4}$.	**15.** $\frac{1}{2} - \frac{1}{9}$.	**20.** $\frac{1}{5} - \frac{1}{13}$.

3. MULTIPLICATION: WHOLE NUMBERS, FRACTIONS, AND DECIMALS

When the multiplier is a whole number, **multiplication** is a short form of finding a number equal in value to the sum of a number of equal addends.

$$\text{Thus, } 5 \times \$7 = \$7 + \$7 + \$7 + \$7 + \$7 = \$35.$$
$$5 \times .07 = .07 + .07 + .07 + .07 + .07 = .35.$$
$$5 \times \tfrac{7}{8} = \tfrac{7}{8} + \tfrac{7}{8} + \tfrac{7}{8} + \tfrac{7}{8} + \tfrac{7}{8} = \tfrac{35}{8} = 4\tfrac{3}{8}.$$

The 5 in each example given is the **multiplier**; \$7, .07, and $\frac{7}{8}$ **are the multiplicands**; and \$35, .35, and $\frac{35}{8}$ are the **products.**

From the meaning of multiplying by a whole number, it follows that:

1. *The multiplier must be an abstract number.*
2. *The multiplicand may be either abstract or concrete.*

3. *The product must be a number of the same name or kind as the multiplicand.*

From principle 3 above, it follows that when the multiplier is a whole number, there will be as many decimal places in the product as there are in the multiplicand.

It also follows that a fraction is multiplied by a whole number by multiplying the numerator and leaving the denominator unchanged.

Check multiplication by going over the work a second time.

Drill Exercises

1. 305×475.	8. 916×70.9.	15. $9 \times \frac{7}{8}$.
2. 906×56.8.	9. 426×90.3.	16. $8 \times \frac{2}{9}$.
3. 506×39.2.	10. 481×7.06.	17. $10 \times \frac{5}{7}$.
4. 275×3.09.	11. $4 \times \frac{3}{5}$.	18. $12 \times \frac{3}{5}$.
5. 460×52.8.	12. $5 \times \frac{3}{4}$.	19. $14 \times \frac{5}{9}$.
6. 326×84.3.	13. $7 \times \frac{2}{3}$.	20. $16 \times \frac{2}{3}$.
7. $506 \times .628$.	14. $6 \times \frac{4}{5}$.	21. $19 \times \frac{2}{5}$.

Multiplication by Fractions and by Decimals

Multiplying by a fraction is both a multiplication and a division. The numerator is the multiplier and the denominator is the divisor.

Thus, $\frac{3}{4} \times 24 = 3 \times 24 \div 4$; $.4 \times 5.3 = 4 \times 5.3 \div 10$.

To multiply a fraction by a fraction, take the product of the numerators for the numerator of the product and the product of the denominators for the denominator of the product.

Work is saved by **cancelling** factors that occur in both terms, before multiplying.

In the product of two decimals, there are as many digits at the right of the decimal point as there are in the total number at the right of the decimal point in both multiplier and multiplicand.

Drill Exercises

Find the products:

1. $\frac{3}{4} \times \frac{12}{13}$.
2. $\frac{5}{6} \times \frac{7}{15}$.
3. $\frac{7}{8} \times \frac{16}{17}$.
4. $\frac{2}{3} \times \frac{7}{8}$.
5. $\frac{4}{5} \times \frac{10}{11}$.
6. $\frac{8}{9} \times \frac{27}{32}$.
7. $\frac{7}{8} \times \frac{16}{21}$.
8. $\frac{3}{5} \times \frac{15}{16}$.
9. $\frac{7}{9} \times \frac{18}{35}$.
10. $\frac{5}{6} \times \frac{24}{25}$.
11. $\frac{11}{12} \times \frac{36}{55}$.
12. $\frac{8}{9} \times \frac{63}{64}$.
13. $\frac{8}{11} \times \frac{22}{25}$.
14. $\frac{15}{16} \times \frac{32}{45}$.
15. $\frac{14}{15} \times \frac{25}{28}$.
16. $\frac{13}{14} \times \frac{28}{39}$.
17. $\frac{15}{16} \times \frac{48}{51}$.
18. $\frac{14}{17} \times \frac{34}{49}$.
19. $\frac{11}{12} \times \frac{48}{77}$.
20. $\frac{13}{15} \times \frac{45}{78}$.
21. 1.75×36.9.
22. 3.45×16.3.
23. 40.3×5.65.
24. 16.5×2.85.
25. 19.3×40.6.
26. 57.3×30.3.
27. 42.1×64.9.
28. 5.06×84.7.
29. 39.2×9.03.
30. 58.2×10.8.

Finding a Per Cent of a Number

Per cent is only another name and notation for **hundredths**. Thus, 7% of $480 = .07 \times 480$; $2\frac{1}{2}\%$ of $360 = .025 \times 360$.

Change to decimals:

1. 45%.
2. 82%.
3. 156%.
4. 245%.
5. 300%.
6. $12\frac{1}{2}\%$.
7. $14\frac{1}{2}\%$.
8. $26\frac{1}{4}\%$.
9. $11\frac{3}{4}\%$.
10. $5\frac{1}{2}\%$.
11. 200%.
12. 550%.
13. 725%.
14. 1100%.
15. 14.5%.
16. 4.5%.
17. 13.25%.
18. $.84\%$.
19. $.16\%$.
20. 1.25%.

Find:

21. 28 % of 456.
22. 14 % of 96.8.
23. 3.5 % of 1650.
24. $4\frac{1}{2}$ % of 846.
25. 17.25 % of 960.
26. $8\frac{1}{4}$ % of 1280.
27. 8.2 % of 34.8.
28. 9.3 % of 168.4.
29. $10\frac{1}{2}$ % of 86.3.
30. $18\frac{1}{4}$ % of 1600.
31. 2.48 % of 1940.
32. 7.8 % of 3680.
33. 340 % of 240.
34. 156 % of 390.
35. 285 % of 346.
36. 178 % of 1750.
37. 204.5 % of 34,200.
38. 196.5 % of 17,500.

Multiplying by Powers of 10

Always multiply by any power of 10, as 10, 100, 1000, etc., by annexing zeros to a whole number, or moving the decimal point in decimals. For either has the effect of moving the digits to higher orders. Thus,

$100 \times 75 = 7500$; $1000 \times 845 = 845{,}000$; $10 \times 387 = 3870$;
$100 \times 6.84 = 684$; $100 \times 1.756 = 175.6$; $1000 \times 8.45 = 8450$.

Drill Exercises

Give products at sight:

1. 10×85.
2. 100×64.
3. 100×7.56.
4. 100×8.3.
5. 100×17.365.
6. $100 \times .048$.
7. 10×1.768.
8. $100 \times .165$.
9. 1000×3.5.
10. $1000 \times .48$.
11. $100 \times .1635$.
12. 1000×8.3.
13. $1000 \times .046$.
14. $1000 \times .0478$.
15. 1000×8.64.

Multiplying by Multiples of Powers of 10

When both factors end in zeros, work is saved as follows: $500 \times 15{,}000 = 5 \times 15$ with five zeros annexed.

$400 \times 600 = 240{,}000$; $700 \times 1300 = 910{,}000$; $500 \times 1300 = 650{,}000$.

Drill Exercises

At sight give the products:

1. 30×80.	**6.** 90×800.	**11.** 60×130.	**16.** 50×160.
2. 40×300.	**7.** 80×600.	**12.** 70×120.	**17.** 40×150.
3. 50×70.	**8.** 70×900.	**13.** 80×160.	**18.** 60×150.
4. 60×800.	**9.** 30×150.	**14.** 90×700.	**19.** 80×200.
5. 300×40.	**10.** 40×160.	**15.** 60×900.	**20.** 90×120.

21. Find the product of 2800×3600.

WORK

$$\begin{array}{r}
3600 \\
\underline{2800} \\
288 \\
\underline{72} \\
10{,}080{,}000
\end{array}$$

EXPLANATION. — Only 36 and 28 were used in the actual multiplication. When this product (1008) was found, four zeros were annexed.

Find the products:

22. 170×3600.	**25.** 340×860.	**28.** 420×8200.
23. 420×5400.	**26.** 250×9800.	**29.** 350×8160.
24. 160×8400.	**27.** 320×8520.	**30.** 230×7620.

Multiplying by Aliquot Parts of 10 or 100

The **aliquot part** of a number is a number that is contained in it an integral number of times. Aliquot parts of 10 and 100 are so important that they should be memorized.

TABLE OF ALIQUOT PARTS

$5 = \frac{1}{2}$ of 10	$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100
$2\frac{1}{2} = \frac{1}{4}$ of 10	$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100
$3\frac{1}{3} = \frac{1}{3}$ of 10	$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100

Tell the reason for the following:

$2\frac{1}{2} \times 32 = \frac{320}{4} = 80$; $3\frac{1}{3} \times 27 = \frac{270}{3} = 90$.

$25 \times 42 = \frac{4200}{4} = 1050$; $33\frac{1}{3} \times 38 = \frac{3800}{3} = 1266\frac{2}{3}$.

Drill Exercises

Find as above:

1. 5×846.	7. 50×865.	13. $33\frac{1}{3} \times 248$.
2. $2\frac{1}{2} \times 936$.	8. 25×932.	14. $16\frac{2}{3} \times 765$.
3. $3\frac{1}{3} \times 729$.	9. $12\frac{1}{2} \times 864$.	15. $8\frac{1}{3} \times 896$.
4. 5×1750.	10. 50×753.	16. $33\frac{1}{3} \times 576$.
5. $2\frac{1}{2} \times 1340$.	11. 25×875.	17. $16\frac{2}{3} \times 645$.
6. $3\frac{1}{3} \times 1650$.	12. $12\frac{1}{2} \times 932$.	18. $8\frac{1}{3} \times 763$.

Multiplying by Special Per Cents

Certain per cents are more easily used when changed to their fractional equivalents. They are:

$50\% = \frac{1}{2}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$25\% = \frac{1}{4}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$

Drill Exercises

Give at sight:

1. 50% of 84.	5. $33\frac{1}{3}\%$ of 63.	9. $16\frac{2}{3}\%$ of 180.
2. 25% of 120.	6. $16\frac{2}{3}\%$ of 36.	10. $16\frac{2}{3}\%$ of 240.
3. $12\frac{1}{2}\%$ of 96.	7. 25% of 128.	11. $33\frac{1}{3}\%$ of 210.
4. 50% of 420.	8. $12\frac{1}{2}\%$ of 168.	12. 50% of 750.

Making Use of Known Products

The following example shows how to make use of a known product to save work.

Find 287×375.

WORK

$$\begin{array}{r} 375 \\ \underline{287} \\ 2625 \\ \underline{10500} \\ 107625 \end{array}$$

EXPLANATION. — When 7×375, or 2625, is known, 280×375 can be found by finding 40×2625, for this is $40 \times 7 \times 375$, or 280×375.

Drill Exercises

Find as above:

1. 217×624.
2. 328×725.
3. 426×864.
4. 357×796.
5. 549×834.
6. 637×528.
7. 459×826.
8. 369×768.
9. 567×936.
10. 248×793.
11. 324×842.
12. 728×966.
13. 546×726.
14. 648×584.
15. 427×645.
16. 355×846.
17. 155×964.
18. 287×839.

4. DIVISION: WHOLE NUMBERS, FRACTIONS, AND DECIMALS

For a discussion of the meanings of division, see **Book I.** Work is given here for practice and for short methods.

Check division by going over the work a second time.

Drill Exercises

Find and check:

1. $10{,}635 \div 36$.
2. $42{,}738 \div 63$.
3. $90{,}684 \div 97$.
4. $35{,}680 \div 2.96$.
5. $42{,}340 \div 36.1$.
6. $87.35 \div 2.46$.
7. $96.3 \div 1.76$.
8. $8.361 \div .197$.
9. $19.34 \div .946$.
10. $356.2 \div 42.5$.
11. $42.6 \div 17.34$.
12. $3.98 \div .063$.
13. $17.02 \div .098$.
14. $64.3 \div .185$.
15. $4.63 \div .028$.

16. $\frac{3}{4} \div \frac{5}{6}$.

17. $\frac{2}{3} \div \frac{5}{7}$.

18. $\frac{5}{6} \div \frac{2}{5}$.

19. $\frac{7}{8} \div \frac{2}{3}$.

20. $\frac{9}{10} \div \frac{3}{4}$.

21. $\frac{11}{12} \div \frac{2}{5}$.

22. $\frac{14}{15} \div \frac{3}{5}$.

23. $\frac{15}{16} \div \frac{7}{8}$.

24. $\frac{12}{13} \div \frac{3}{8}$.

25. Divide $368\frac{2}{3}$ by 7.

WORK

$7\overline{)368\frac{2}{3}}$

$52,\quad 4\frac{2}{3}$ remainder

$\frac{1}{7} \times \frac{14}{3} = \frac{2}{3}$

Hence, $368\frac{2}{3} \div 7 = 52\frac{2}{3}$.

EXPLANATION.— Divide as in whole numbers until the remainder is less than the divisor, then divide the fraction or mixed number as shown here.

Divide and check:

26. $739\frac{1}{3} \div 8$.

27. $576\frac{2}{3} \div 7$.

28. $863\frac{3}{4} \div 8$.

29. $927\frac{1}{2} \div 5$.

30. $648\frac{2}{3} \div 7$.

31. $596\frac{3}{4} \div 8$.

32. $725\frac{1}{4} \div 9$.

33. $697\frac{2}{5} \div 6$.

34. $816\frac{2}{5} \div 6$

35. $597\frac{2}{7} \div 9$.

36. $572\frac{2}{3} \div 8$.

37. $673\frac{3}{4} \div 7$.

Dividing by Numbers Ending in Zeros

Before dividing, all zeros should be cut off the divisor and a corresponding change made in the dividend. To divide 1356.48 by 2600, the work should be:

WORK

$$\begin{array}{r} .52\,17 \\ 26\cancel{0}\cancel{0}\overline{)13_{\wedge}56.48} \\ \underline{13\;0} \\ 56 \\ \underline{52} \\ 44 \\ \underline{26} \\ 188 \\ \underline{182} \end{array}$$

EXPLANATION. — Since the divisor was divided by 100 by cutting off the two zeros, the dividend was also divided by 100 by moving the decimal point two places to the left. This follows the principle that dividing both dividend and divisor by the same number does not affect the quotient.

Drill Exercises

Divide as above:

1. $4685 \div 200$.
2. $6736 \div 300$.
3. $783.6 \div 400$.
4. $875.8 \div 900$.
5. $658.7 \div 700$.
6. $9687 \div 2000$.
7. $8765 \div 5000$.
8. $9847 \div 4000$.
9. $8645 \div 4800$.
10. $7230 \div 5400$.
11. $6950 \div 6300$.
12. $746.8 \div 3500$.
13. $693.7 \div 2800$.
14. $564.8 \div 3200$.
15. $76.84 \div 540$.
16. $70.65 \div 610$.
17. $36.84 \div 400$.
18. $9.68 \div 500$.
19. $17.35 \div 600$.
20. $8.3 \div 700$.
21. $16.2 \div 500$.
22. $4.26 \div 800$.
23. $5.6 \div 700$.
24. $6.4 \div 800$.

Dividing by Aliquot Parts of 10 or 100

The use of aliquot parts of 10 and 100 is shown by the following problems. Since $2\frac{1}{2} = \frac{10}{4}$, $32 \div 2\frac{1}{2} = \frac{4}{10} \times 32 = 12.8$; since $33\frac{1}{3} = \frac{100}{3}$, $48 \div 33\frac{1}{3} = \frac{3}{100} \times 48 = 1.44$.

The work can be done without a pencil. Thus,

$$1348 \div 25 = 4 \times 13.48 = 53.92;$$
$$1625 \div 33\tfrac{1}{3} = 3 \times 16.25 = 48.75;$$
$$3852 \div 16\tfrac{2}{3} = 6 \times 38.52 = 231.12.$$

Drill Exercises

Without a pencil find:

1. $3846 \div 25$.
2. $1697 \div 33\frac{1}{3}$.
3. $2468 \div 50$.
4. $2173 \div 16\frac{2}{3}$.
5. $4263 \div 50$.
6. $1968 \div 25$.
7. $2145 \div 12\frac{1}{2}$.
8. $3692 \div 25$.
9. $1698 \div 33\frac{1}{3}$.
10. $2165 \div 12\frac{1}{2}$.
11. $1168 \div 16\frac{2}{3}$.
12. $2046 \div 25$.
13. $13.48 \div 50$.
14. $241.5 \div 25$.
15. $165.8 \div 12\frac{1}{2}$.
16. $21.68 \div 16\frac{2}{3}$.
17. $75.16 \div 33\frac{1}{3}$.
18. $124.6 \div 25$.
19. $216.4 \div 50$.
20. $192.8 \div 25$.
21. $92.68 \div 25$.

When Both Multiplication and Division Occur

When both multiplication and division occur, common factors should be removed. Thus, $16 \times 345 \div 32 = 345 \div 2$; $24 \times 275 \div 16 = 3 \times 275 \div 2$; $328 \times 16 \div 8 = 2 \times 328$.

Drill Exercises

At sight give:

1. $14 \times 284 \div 28$.
2. $26 \times 125 \div 13$.
3. $24 \times 213 \div 8$.
4. $7 \times 456 \div 21$.
5. $9 \times 378 \div 27$.
6. $27 \times 84 \div 9$.
7. $35 \times 96 \div 7$.
8. $7 \times 125 \div 35$.
9. $48 \times 96 \div 8$.
10. $8 \times 150 \div 48$.
11. $9 \times 135 \div 54$.
12. $8 \times 245 \div 56$.
13. $56 \times 72 \div 8$.
14. $54 \times 86 \div 9$.
15. $72 \times 68 \div 8$.

Miscellaneous Drill

At sight give:

1. 90×120.
2. 70×800.
3. 80×900.
4. 5×842.
5. $2\frac{1}{2} \times 968$.
6. $3\frac{1}{3} \times 735$.
7. 25×840.
8. 50×348.
9. $12\frac{1}{2} \times 368$.
10. $33\frac{1}{3} \times 564$.
11. $16\frac{2}{3} \times 456$.
12. $8\frac{1}{3} \times 560$.
13. $48 + 56$.
14. $93 + 49$.
15. $74 + 38$.
16. $96 + 37$.
17. $58 + 96$.
18. $87 + 56$.
19. $78 + 47$.
20. $93 + 86$.
21. $54 + 73$.
22. $49 + 56$.
23. $75 + 67$.
24. $53 + 84$.
25. 50 % of 96.
26. 25 % of 72.
27. $33\frac{1}{3}$ % of 54.
28. 50 % of 75.
29. 25 % of 38.
30. $12\frac{1}{2}$ % of 25.
31. $16\frac{2}{3}$ % of 48.
32. $33\frac{1}{3}$ % of 58.
33. 75 % of 36.
34. $66\frac{2}{3}$ % of 18.
35. $12\frac{1}{2}$ % of 85.
36. $33\frac{1}{3}$ % of 84.

CHAPTER II

THE FORMULA

In **Book I** you saw that the principles of mensuration were expressed in a kind of shorthand, using letters instead of words. Thus, instead of saying that "the number of square units in the area of any rectangle is the product of the number of linear units in its two dimensions," you simply said

$$A = bh.$$

This shorthand expression is called a **formula.** This convenient form of expressing mathematical relations is used in various kinds of industrial and commercial work. You will find formulæ used in science, in trade journals, in books on mechanics, and in various articles that you will read.

*State in words the principles expressed by the following formulæ, used in mensuration in **Book I**:*

1. $A = bh.$	4. $A = \frac{1}{2}\,cr.$	7. $V = Bh.$
2. $A = \frac{bh}{2}.$	5. $A = \pi r^2.$	8. $V = \pi r^2 h.$
3. $A = \frac{h(b + b')}{2}.$	6. $V = abc.$	9. $V = \frac{\pi d^2 h}{4}.$

1. EVALUATING FORMULÆ

To **evaluate** a formula is to substitute the numbers represented by the letters and then perform the computation. Thus, in the formula $V = abc$, to find V when $a = 5$, $b = 6$, and $c = 8$, $V = 5 \times 6 \times 8 = 240$.

1. Find A when $b = 5$ and $h = 6$ in the formula $A = bh$.

2. Find A when $h=6$, $b=14$, and $b'=8$ in the formula $A=\frac{h(b+b')}{2}$.

3. The area of a circle is expressed by the formula $A=\pi r^2$, where $\pi=3.1416$ and r the radius. Find the area of a circle whose radius is 12 ft.

4. The volume of a right circular cylinder is expressed by the formula $V=\pi r^2 h$. Find the volume of a cylinder whose height is 12 inches and the radius of whose base is 5 inches.

5. Another formula for the volume of a cylinder is $V=\frac{\pi d^2 h}{4}$, where d represents the diameter. Find the volume of a cylinder whose height is 12 ft. and whose diameter is 10 ft., first using one of the formulæ, and then checking by using the other.

6. If c represents the circumference of a circle, and d its diameter, express the formula for c in terms of d.

2. SIMPLIFYING LITERAL EXPRESSIONS

When mathematical principles or relations are expressed by letters, the expression is called a **literal** or an **algebraic expression.** It is often necessary to simplify these expressions.

Addition

Just as 3 lb. + 5 lb. + 2 lb. = 10 lb., so $3a+5a+2a=10a$ where a represents any value whatever.

At sight give the sums:

1.	2.	3.	4.	5.
$3a$	$4c$	$7m$	$8x$	$7y$
$5a$	$2c$	$2m$	$2x$	$2y$
$7a$	$6c$	$4m$	$9x$	$3y$
$6a$	$8c$	$2m$	$4x$	$5y$

6. Just as $3a + 5a = 8a$, so $3 \times 65 + 5 \times 65 = 8 \times 65 = 520$; and $7 \times 428 + 3 \times 428 = 10 \times 428 = 4280$.

At sight give the results:

7. $8 \times 248 + 2 \times 248$.
8. $6 \times 785 + 4 \times 785$.
9. $7 \times 3.1416 + 3 \times 3.1416$.
10. $4 \times 896 + 16 \times 896$.
11. $12 \times 565 + 8 \times 565$.
12. $17 \times 350 + 13 \times 350$.
13. $18 \times 620 + 12 \times 620$.
14. $84 \times 37.5 + 16 \times 37.5$.
15. $48 \times 7.26 + 52 \times 7.26$.
16. $35 \times 9.87 + 65 \times 9.87$.

Subtraction

Just as 8 lb. – 3 lb. = 5 lb., so $8a - 3a = 5a$, where a represents any value whatever.

At sight give the differences:

1.	2.	3.	4.	5.
$9a$	$11b$	$13x$	$16y$	$12c$
$\underline{2a}$	$\underline{6b}$	$\underline{5x}$	$\underline{9y}$	$\underline{7c}$

6. Just as $13a - 8a = 5a$, so $13 \times 380 - 8 \times 380 = 5 \times 380 = 1900$.

At sight give the results:

7. $17 \times 275 - 7 \times 275$.
8. $27 \times 320 - 22 \times 320$.
9. $16 \times 480 - 12 \times 480$.
10. $48 \times 750 - 45 \times 750$.
11. $26 \times 975 - 16 \times 975$.
12. $42 \times 982 - 32 \times 982$.
13. $13 \times 640 - 8 \times 640$.
14. $26 \times 820 - 21 \times 820$.
15. $19 \times 720 - 16 \times 720$.
16. $98 \times 380 - 88 \times 380$.

Multiplication

The expression aa or $a \times a$ is simplified by writing it a^2, read "a square." Likewise, bbb or $b \times b \times b$ is written b^3, read "b cube"; and $cccc$ is written c^4, read "c to the fourth power."

$2a \times 3a$ is simplified to $6a^2$; $2a \times 4b$, to $8ab$.

Simplify at sight:

1. $3b \times 4b$.
2. $6a \times 3a$.
3. $7y \times 3y$.
4. $9c \times 4c$.
5. $8b \times 4b$.
6. $7c \times 8c$.
7. $9d \times 7d$.
8. $8r \times 5r$.
9. $2a \times 3a \times 5a$.
10. $4a \times 2a \times 5a$.
11. $2b \times 4b \times 10b$.
12. $5c \times 2c \times 8c$.

13. $3a \times 2b$ is simplified to $6ab$. Simplify $5b \times 6c$. Simplify $3r \times 6s$; $5a \times 6t$; $7m \times 6n$.

At sight give the simplest forms of:

14. $5a \times 7b$.
15. $6b \times 7c$.
16. $9d \times 6e$.
17. $8a \times 7c$.
18. $7b \times 6d$.
19. $9a \times 6d$.
20. $7d \times 8e$.
21. $6c \times 8b$.
22. $9e \times 8g$.

Division

Since division is the inverse of multiplication, $6a^2 \div 3a = 2a$; $8ab \div 2a = 4b$; $16de \div 8 = 2de$.

At sight give:

1. $8ab \div 4b$.
2. $9ac \div 3a$.
3. $16a^2 \div 2a$.
4. $21cd \div 7c$.
5. $48xy \div 6x$.
6. $54b^2 \div 6b$.
7. $72ac \div 9a$.
8. $48c^2 \div 6c$.
9. $56c^3 \div 7c$.
10. $63d^3 \div 9d$.
11. $42abc \div 6c$.
12. $32acd \div 8cd$.
13. $45bcd \div 5bc$.
14. $54xyz \div 6xy$.
15. $48a^2b \div 6a^2$.

16. This work is just what you have done in division of arithmetical numbers. That is, it is merely canceling like factors from both dividend and divisor. Thus, $16\pi \div 2\pi = 8$; $48\pi \div 16\pi = 3$.

At sight give:

17. $8 \times 9 \div 2 \times 9$.

18. $16 \times 7 \div 4 \times 7$.

19. $20 \times 17 \div 5 \times 17$.

20. $48 \times 37 \div 8 \times 37$.

21. $54 \times 350 \div 9 \times 350$.

22. $63 \times 480 \div 7 \times 480$.

23. $49 \times 365 \div 7 \times 365$.

24. $72 \times 296 \div 9 \times 296$.

25. $45 \times 19 \div 5 \times 19$.

26. $36 \times 24 \div 4 \times 24$.

27. $56 \times 52 \div 8 \times 52$.

28. $64 \times 87 \div 8 \times 87$.

3. FACTORING A FORMULA

Just as in $5a + 3a$ the unlike factors 5 and 3 are added, so in $5a+5b$ the unlike factors a and b may be added. That is, $5a + 5b = 5(a + b)$. Thus, if $a = 4$ and $b = 6$, 4 and 6 may be added before multiplying; thus making but one multiplication instead of two.

When $5a + 5b$ is changed into $5(a + b)$, the expression is said to be **factored**, for it is changed to the product of two factors.

Represent as two factors:

1. $3a + 3b$.

2. $7b + 7c$.

3. $5c + 5d$.

4. $8a + 8c$.

5. $7a + 7d$.

6. $10x + 10y$.

7. $9a + 9b$.

8. $2a^2 + 2b^2$.

9. $3ab + 3cd$.

10. $5ab^2 + 5c^2$.

11. $6ab + 6d$.

12. $4xy + 4ab$.

Give the value of:

13. $7 \times 35 + 7 \times 65$.

14. $9 \times 25 + 9 \times 75$.

15. $8 \times 36 + 8 \times 64$.

16. $8 \times 17 + 8 \times 33$.

17. $9 \times 21 + 9 \times 29$.

18. $7 \times 83 + 7 \times 17$.

4. FORMULÆ DERIVED FROM OTHER FORMULÆ

When certain fundamental formulæ are known, others needed may be derived from these. Examples to show how this is done are given here.

1. You know that the area of a rectangle is represented by $A = bh$, where A represents the number of square units in the area, and b and h represent the number of linear units in the base and height, respectively. From this we know that $A \div b = h$ or $A \div h = b$. Instead of the division sign, these are usually written

$$h = \frac{A}{b} \text{ and } b = \frac{A}{h}.$$

This result is evident from the meaning of division. That is, the product of two factors divided by either gives the other.

2. From the formula $c = \pi d$, give a formula for d in terms of π and c.

3. From the formula $V = Bh$, give the value of B in terms of of V and h. Of h, in terms of V and B.

4. From the formula derived in problem 2, find the diameter of a circle whose circumference is 150 ft.

5. From the first formula derived in problem 3, find the area of the base of a prism whose volume is 600 cu. in. and whose height is 20 in. Find the height of a prism whose volume is 500 cu. in. and the area of whose base is 25 sq. in.

6. From the formula $c = 2\pi r$, give the value of r in terms of c and 2π.

7. From the formula $A = \frac{bh}{2}$, it is evident that $2A = bh$. From $A = \frac{1}{2}h(b + b')$, give the value of $2A$.

8. From the formula $A = \frac{bh}{2}$, derive a formula for b, and one for h.

9. When the area of a triangle is 64 sq. in. and the base 16 in., what is the altitude?

10. In the formula $V = abc$, give a in terms of V, b, and c.

11. In the formula $A = \frac{1}{2}h(b + b')$, give h in terms of the other letters.

12. How high must a trapezoid be whose bases are 8 in. and 12 in., respectively, if the area is 60 sq. in.?

13. To hold 15 tons of coal when filled to a depth of 5 ft., how many square feet must there be in the floor of the bin? (1 cu. ft. of coal = 65 lbs.)

14. To hold 15 tons of coal, a bin 8 ft. by 10 ft. will have to be filled to what depth?

15. The volume of a cylinder is expressed by the formula, $V = \pi r^2 h$. To what depth will 5 gal. of milk fill a milk can 14 in. in diameter?

16. A circular running track $\frac{1}{8}$ mi. (660 ft.) around has a diameter of how many feet?

17. To what depth must a box 30 in. by 42 in. be filled to hold 5 bu.? (1 bu. = 2150.42 cu. in.)

18. A garden 120 ft. long must be how wide to contain the same area as a garden 85 ft. by 96 ft.?

19. A triangle with a base of 24 in. must have what altitude to contain 192 sq. in.?

CHAPTER III

THE EQUATION

AN **equation** is a statement that two expressions are equal, or that they have the same value. The formulæ which you have studied were equations. So are such expressions as

$$2 \times 12 = 4 \times 6 \text{ and } 3n = 15.$$

In reducing formulæ to other forms, and in solving many problems that arise in mathematics, use is made of the equation.

In solving any problem, we are seeking an **unknown value.** To solve a problem by use of an equation, the unknown value is expressed by some letter and the relation of the known to the unknown is expressed by an equation.

Thus, in the problem, "What number added to 15 gives a sum of 21?" the relation may be expressed

$$n + 15 = 21, \textit{ an equation.}$$

It is evident that $n = 6$, for 6 is the only number which added to 15 gives 21.

The expression on each side of the sign of equality is a **member** of the equation.

Such problems as these are solved as easily without the use of the equation, but the illustration is given merely to furnish a simple example of how an equation may be used in the solution of a problem.

1. DETERMINING THE VALUE OF THE UNKNOWN NUMBER

To find a value of the unknown number that makes both sides equal, or satisfies the equation, is to **solve** the equation. By inspection you can solve such equations as $x + 2 = 6$, for this merely asks, "What number added to 2 equals 6?" and you know that it is 4, for $4 + 2 = 6$.

At sight solve:

1. $x + 3 = 9$.	6. $7 + x = 12$.	11. $n + 7 = 14$.
2. $x + 6 = 10$.	7. $8 + x = 14$.	12. $8 + s = 22$.
3. $x + 5 = 20$.	8. $11 + x = 20$.	13. $9 + r = 17$.
4. $x + 7 = 18$.	9. $12 + x = 30$.	14. $t + 6 = 15$.
5. $x + 9 = 14$.	10. $15 + x = 40$.	15. $3 + n = 20$.

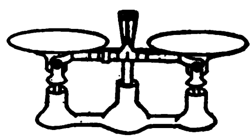
Equations Requiring Subtraction

In the above equations, you found the value of the unknown by subtraction. Thus, in $x + 3 = 9$, to find what number added to 3 equals 9, you subtracted 3 from 9.

In more complicated equations that arise, use is made of an evident truth, called an **axiom**. This is stated thus,

If equals are subtracted from equals, the remainders are equal.

This is too evident to be questioned, but an illustration is of interest. An equation is really an expression of the balance of values and may thus be compared to the balance used in weighing.



Each member of the equation corresponds to the weights in the two pans. If the weights in either pan are changed, there must be a corresponding change in the other.

Thus, in the equation $x + 8 = 12$, if each member represents weights in the pans of the balance, and 8 is taken from one pan, it must be taken from the other also.

Hence, from $x + 8 = 12$
we have $x = 12 - 8$; (8 being subtracted from each member)
hence, $x = 4$, *the solution.*

The solution is **checked** by substituting the value found and seeing if it satisfies the equation. Thus, $4 + 8 = 12$ satisfies the equation, and the solution is correct.

Solve:

1. $n + 4 = 9$.
2. $x + 6 = 15$.
3. $x + 12 = 38$.
4. $3x = 2x + 7$.
5. $5x = 4x + 9$.
6. $8 + x = 15.2$.
7. $2.5 + x = 9.75$.
8. $x + \frac{3}{4} = 2\frac{1}{2}$.
9. $x + 2.8 = 7.63$.
10. $y + 1.09 = 6.81$.
11. $2\frac{1}{2} + x = 7\frac{1}{2}$.
12. $3\frac{1}{4} + x = 9\frac{5}{8}$.
13. $1.5 + x = 16.25$.
14. $3.4 + x = 9.8$.
15. $7.3 + x = 16$.

Equations Requiring Division

The solution of a problem may require such an equation as $3x + 5 = 17$. Here we see by subtraction that $3x = 12$. The question here is, "What number multiplied by 3 equals 12?" From the meaning of division we see the answer is 4. But, using the balance again to illustrate the two members of an equation, we see that if one member is divided the other member must also be divided by the same number. This is expressed as an axiom by saying,

If equal numbers are divided by equal numbers (not zeros), the quotients are equal.

Thus, if $3x = 12$,
then $x = 12 \div 3 = 4$, by dividing each member by 3.

Solve:

1. $3x = 15.$	6. $3x + 7 = 25.$	11. $5x + 7 = 42.$
2. $7x = 42.$	7. $5x + 2 = 32.$	12. $6x + 8 = 50.$
3. $9x = 36.$	8. $7x + 5 = 40.$	13. $7x + 10 = 66.$
4. $12x = 72.$	9. $8x + 2 = 18.$	14. $8x + 9 = 65.$
5. $8x = 56.$	10. $9x + 7 = 70.$	15. $7x + 15 = 36.$

Equations Requiring Addition

The solution of a problem may require such an equation as $x - 5 = 20$. The question here is, "From what number may 5 be subtracted and leave 20?" The answer is evidently 25. But since the expression $x - 5$ is 5 less than x, 5 would have to be added to it to get x. That is, $x - 5 + 5 = x$. Using the balance again as an illustration, 5 was added to each member without affecting the equation. Stated as an axiom,

If equals are added to equals, the sums are equal.

Thus, if $x - 5 = 20$,

then $x = 20 + 5$, by adding 5 to each member.

Solve:

1. $x - 3 = 8.$	6. $x - 48 = 125.$	11. $3x - 9 = 6.$
2. $x - 7 = 15.$	7. $x - 96 = 84.$	12. $4x - 10 = 18.$
3. $x - 3 = 2.$	8. $x - 8.4 = 14.2.$	13. $5x - 8 = 17.$
4. $x - 4 = 7.$	9. $x - 3.5 = 9.6.$	14. $3x - 90 = 90.$
5. $x - 6 = 10.$	10. $x - 1.8 = 2.1.$	15. $4x - 60 = 100.$

Equations Solved by Multiplication

The solution of a problem may require such an equation as $\frac{x}{3} = 7$. The question here is, "One-third of what number is 7?" The answer is evidently 21. While you could have

answered this simple question by trial or inspection, it is usually solved by multiplying both members by 3. The authority for this, expressed as an axiom, is,

If equals are multiplied by equals, the products are equal.

Thus, if $\frac{x}{3} = 7,$

then $x = 3 \times 7$, by multiplying both members by 3.

Solve:

1. $\frac{x}{2} = 8.$
2. $\frac{x}{3} = 9.$
3. $\frac{x}{5} = 7.$
4. $\frac{x}{4} = 12.$
5. $\frac{x}{6} = 9.$
6. $\frac{x}{2} = 3.5.$
7. $\frac{x}{4} = 2.4.$
8. $\frac{x}{8} = 2.5.$
9. $\frac{x}{7} = 3.4.$
10. $\frac{x}{9} = 1.2.$
11. $\frac{x}{2} = 3\frac{3}{4}.$
12. $\frac{x}{4} = \frac{2}{3}.$
13. $\frac{x}{7} = \frac{5}{6}.$
14. $\frac{x}{8} = \frac{2}{3}.$
15. $\frac{x}{10} = \frac{3}{5}.$

Miscellaneous Exercises for Drill

Solve:

1. $x + 3.5 = 8.$
2. $x - 5 = 12.$
3. $2x + 6 = 28.$
4. $4x - 1 = 23.$
5. $x + 2\frac{1}{2} = 7.$
6. $\frac{3}{4}x = 9.$
7. $\frac{x}{7} = \frac{2}{3}.$
8. $5x - 3 = 42.$
9. $x + 3.1 = 9.6.$
10. $3x + 7 = 34.$
11. $x - 3.5 = 1.9.$
12. $4x + 3 = 11.8.$
13. $\frac{x}{5} = 1\frac{1}{2}.$
14. $x - 4\frac{1}{2} = 3.$
15. $6x - 7 = 47.$
16. $x + \frac{3}{4} = 7\frac{1}{2}.$
17. $5x + 6 = 46.$
18. $8x - 3\frac{1}{2} = 1\frac{3}{4}.$

2. PROBLEMS SOLVED BY EQUATIONS

You have met no problems in your earlier mathematics that could have been solved more easily by algebra. And, at present, any problem to which you can apply algebra is more in the nature of a puzzle than a problem that meets a real need. To understand some of the work that follows, however, you will need a simple knowledge of equations such as you have gained from studying this chapter. A few problems follow, to give practice in expressing a relation in the form of an equation, and not to meet any real need in life.

1. A line 20 ft. long is to be divided into two parts so that one part is 2 ft. more than twice the other. Find the two lengths.

While this problem may be solved by arithmetic, an algebraic solution is simpler, as shown below.

SOLUTION

Let $x =$ the number of feet in the shorter part; then $2x + 2 =$ the longer part, for this is 2 more than twice x, or the smaller number.

Then $x + 2x + 2$, or $3x + 2 = 20$.

Hence, $3x = 20 - 2 = 18$, subtracting 2 from each member.

Then $x = 18 \div 3 = 6$, dividing each member by 3.

And $2x + 2 = 2 \times 6 + 2 = 14$, substituting 6 for x in the expression for the longer part.

2. Find the lengths into which a rod 16 in. long must be cut so that one piece will be 4 in. longer than the other.

3. In a class of 32 pupils, there are 4 more girls than boys. How many of each are there?

4. If it takes 240 ft. of fencing to inclose a rectangular garden 20 ft. longer than it is wide, how long and how wide is it?

5. The perimeter (distance around) of an isosceles triangle is 56 in. Each of the two equal sides is 4 in. longer than the base. Find the length of the base and of each of the equal sides.

6. Find two consecutive numbers whose sum is 117.

SUGGESTION. — Let $x =$ the smaller of the two numbers.

Then $x + 1 =$ the other, for, being consecutive, it must be 1 larger than the other.

Then $x + x + 1$ or $2x + 1 = 117$. Now solve the equation.

7. Find three consecutive numbers whose sum is 123.

8. Find two consecutive odd numbers whose sum is 56.

SUGGESTION. — Let x be one and $x + 2$ the other. Why?

9. Find two consecutive even numbers whose sum is 98.

10. Two boys together sold a total of 98 papers. One boy sold 12 more than the other. Find how many each boy sold.

11. A woman paid 90¢ for a pound of coffee and a pound of tea. She paid twice as much for the tea as for the coffee. How much did she pay for each?

12. A rectangular lot is twice as long as it is wide. The distance around it is 90 rd. Find its dimensions.

13. Paul and Henry together have 38 marbles. Paul has 10 more than Henry. How many has each?

14. A rectangular lot 40 ft. longer than it is wide is 400 ft. around. Find its dimensions.

15. A boy's salary doubled each year for 3 years. The third year it amounted to $20 per week. What was it the first and second years?

16. When 60 is added to a certain number it gives a number four times the given number. What is the number?

17. When pears cost twice as much as apples, a boy bought 10 of each for 60¢. Find the price of each.

CHAPTER IV

RATIO AND PROPORTION

1. THE MEANING AND USE OF RATIO

WE compare numbers in two ways, either by subtraction or by division. Thus, we say of a number that it is so much larger or so many times as large as another. Thus, 9 is 6 larger than 3, or it is 3 times as large as 3.

When numbers of the same kind are compared by division, the relation is often called a **ratio**. Thus, the ratio of \$12 to \$3 is 4; of 3 ft. to 8 ft., $\frac{3}{8}$; of 5 in. to 2 in., $2\frac{1}{2}$; of 3.8 in. to 4.5 in., $.844^+$; the ratio of the circumference of a circle to its diameter is π; etc. The ratio, then, of one number to a like number is the quotient found by dividing the first by the second. That is, the quotient is the ratio of the dividend to the divisor.

Since the quotient of any number divided by a like number, as feet by feet, or dollars by dollars, is abstract, we see that a ratio is always abstract.

1. What is the ratio of 3 ft. to 6 ft.?

Always give a ratio in its simplest terms. Thus, the above ratio is $\frac{1}{2}$.

2. Give the ratio of \$2.50 to \$7.50; of \$3.50 to \$7; of 6 ft. to $4\frac{1}{2}$ ft.; of 9 mi. to 6 mi.

Give the ratio of:

3. 8 in. to 12 in.

4. 9 ft. to 15 ft.

5. 27 gal. to 18 gal.

6. \$54 to \$45.

7. 300 mi. to 60 mi.
8. 75 yd. to 60 yd.
9. 80 rd. to 140 rd.
10. 700 ft. to 175 ft.
11. \$38.40 to \$17.
12. \$96.50 to \$120.
13. \$8.75 to \$16.30.
14. 9.3 ft. to 16.2 ft.

15. A certain rectangular garden is 60 ft. wide and 80 ft. long. What is the ratio of its width to its length?

16. When a boy sells 48 of 52 papers received, what is the ratio of those sold to those received?

In the past, ratio has often been written with a colon between the two numbers. Thus, the ratio of \$7 to \$12 was written \$7 : \$12 and read "As \$7 is to \$12." While this form is but little used, the expression "as 3 is to 4" and like expressions are still used. Thus, to say that the dimensions of a rectangle are "as 3 is to 4" means that the width is $\frac{3}{4}$ of the length, or that the length is $1\frac{1}{3}$ times the width.

17. The dimensions of a garden are as 2 is to 3. If the garden is 60 ft. wide, how long is it? If it is 80 ft. long, how wide is it?

18. If the dimensions of a page in a book are $7\frac{1}{2}$ in. and $5\frac{1}{4}$ in., what is the ratio of the length to the width? If a page is 10 inches long, how wide must it be to have the same ratio?

19. Construct any two line-segments that are in the ratio of 3 to 4; of 4 to 5.

20. If a man who earns \$25 per week is saving \$10 of it, he is saving what ratio of his earnings?

2. THE MEANING AND USE OF PROPORTION

The equation expressing the equality between two ratios is called a **proportion**. For example, $\frac{\$3}{\$8} = \frac{6 \text{ ft.}}{16 \text{ ft.}}$ is a propor-

tion. It is read "\$3 is to \$8 as 6 ft. is to 16 ft.," or "the ratio of \$3 to \$8 equals the ratio of 6 ft. to 16 ft."

The above proportion was formerly written

$$\$3 : \$8 :: 6 \text{ ft.} : 16 \text{ ft.}$$

But this form is rapidly going out of use. The ratios are considered fractions, and the equality of two ratios is but an equation and follows all the principles of any equation.

Formerly, the subject of proportion was much used in solving many of the problems of everyday life. Now, it is rarely ever used. For example, such a problem as "If 5 acres of potatoes yield 800 bu., how many bushels will 3 acres yield at the same rate?" was solved by proportion. The statement was

$$x : 800 :: 3 : 5$$

That is, the relation or ratio of the yield is the same as the ratio of the areas producing the yield.

This was then solved by the principle that

The product of the extremes (the end terms) is equal to the product of the means (the second and third terms).

This gave the equation $5x = 2400$, from which $x = 480$, by dividing each member by 5.

Even when proportion is now used for such problems, the proportion is stated as an equation as follows:

$$\frac{x}{800} = \frac{3}{5}.$$

Hence, $x = \frac{3 \times 800}{5}$, by multiplying both members by 800.

$x = 480.$

Such problems are not usually solved by proportion. They are usually solved by "unitary analysis" or by the "ratio method." That is, one says, "If 5 acres yield 800 bu.,

1 acre will yield 160 bu. and 3 acres will yield 3×160 bu., or 480 bu." or "The 3-acre field will yield $\frac{3}{5}$ as much, hence, $\frac{3}{5} \times 800$ bu., or 480 bu."

The use of proportion is now confined largely to a method of stating certain geometric and scientific relations or principles.

Proportionality of Areas

1. Compare the areas of two rectangles, one whose dimensions are 6 in. and 10 in., and the other whose dimensions are 8 in. and 15 in.

SOLUTION

$$\frac{\text{Area of first}}{\text{Area of second}} = \frac{6 \times 10}{8 \times 15} = \frac{1}{2}.$$

Therefore their areas are to each other as 1 is to 2.

2. Compare two rectangles whose dimensions are 10 in. and 12 in., and 15 in. and 18 in., respectively.

3. Compare the area A of a rectangle whose dimensions are a and b, with the area A' of a rectangle whose dimensions are a' and b'.

4. The proportion $\frac{A}{A'} = \frac{ab}{a'b'}$, found in problem 3, is stated in words as a principle often used in mathematics as follows:

The areas of two rectangles are to each other as the product of their dimensions.

5. Find the relation of two rectangles having equal bases.

SUGGESTION. — Let a and b be the dimensions of one, and c and b of the other, for the base b is the same in both.

6. The proportion $\frac{A}{A'} = \frac{a}{c}$, found in problem 5, is stated as a principle as follows:

The areas of two rectangles having equal bases are to each other as their altitudes.

7. Find the relation of two rectangles having equal altitudes, and state the result as a principle.

8. Compare two parallelograms having unlike dimensions, as you compared rectangles in problem 3.

State the proportion as a principle.

9. Compare a triangle whose base is 10 ft. and altitude 8 ft., with one whose base is 12 ft. and whose altitude is 12 ft.

10. Compare a triangle whose dimensions are a and b, with one whose dimensions are a' and b'.

11. State the relation $\frac{A}{A'} = \frac{ab}{a'b'}$, which you found in problem 10, as a principle. (See problem 4.)

12. Compare two triangles, each with a base of 10 in., one having an altitude of 7 in., and the other an altitude of 9 in.

13. Compare two triangles with equal bases b, and unequal altitudes a and c, and state the relation as a principle. (See problem 6.)

14. Compare the area of a square whose sides are each 5 in., with one whose sides are each 7 in.

15. Compare the areas of two squares whose sides are respectively a and a' units.

16. The relation $\frac{A}{A'} = \frac{a^2}{a'^2}$ is expressed by a principle as follows:

The areas of two squares are to each other as the squares of their sides.

17. Compare the area of a circle whose radius is 5 in., with one whose radius is 8 in.

$$\frac{\text{First circle}}{\text{Second circle}} = \frac{\pi\, 25 \text{ sq. in.}}{\pi\, 64 \text{ sq. in.}} = \frac{25}{64}.$$

18. Show that the areas of two circles are to each other as the squares of their radii.

19. If one circle has a radius twice that of another, how do their areas compare?

20. If one square has a side three times that of another, how do their areas compare?

21. If the length of a rectangle is made twice as great and its width three times as great, how many times is its area increased?

22. John's garden is twice as long as Frank's but only three fourths as wide. John's garden is how many times as large as Frank's?

23. The carrying capacity of a water pipe depends upon the area of a cross-section of the pipe. A pipe with twice its diameter will discharge how many times as much water?

24. Compare the carrying capacity of a $\frac{1}{2}$-inch pipe with that of a 2-inch pipe.

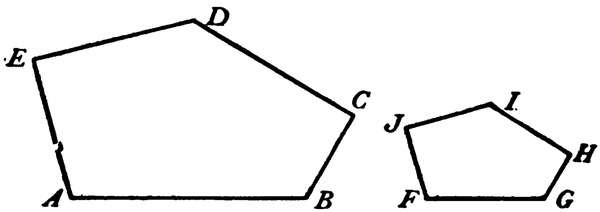
25. If the linoleum for a square floor costs \$40, how much will the same grade of linoleum cost for a square floor but half as long, not considering waste?

CHAPTER V

SIMILAR FIGURES

ANY two figures having the same shape are called **similar figures.** Thus, two squares, two equilateral triangles, or two circles are similar, for they have the same shape.

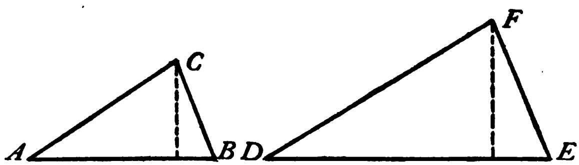
1. FUNDAMENTAL PRINCIPLES OF SIMILAR FIGURES



1. These two polygons are similar. Compare the length of FG with that of AB by the use of your ruler or compasses.

2. Compare all the corresponding sides, as GH with BC, HI with CD, IJ with DE, and JF with EA.

3. In the following similar triangles, compare corresponding sides and the altitudes that are drawn.



If your measurements were carefully made, you found the ratio of corresponding sides in problem 1 to be 2. In problem 3 the ratios were each $1\frac{1}{2}$.

And in general,

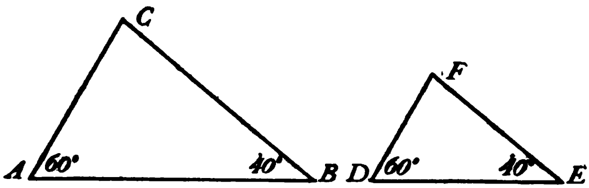
The corresponding lines in similar figures are proportional.

4. Draw any triangle ABC. Now bisecting each side, construct a second triangle with these half sides, making a triangle similar to the first. Measure the corresponding angles of the two triangles.

5. If carefully drawn and carefully measured, the corresponding angles in problem 4 were found to be equal. And in general,

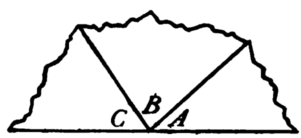
The corresponding angles of similar triangles are equal.

6. Upon any two unequal bases, as AB and DE, construct triangles with base angles 40° and 60°, respectively.



Find the ratio of the corresponding sides. If carefully drawn, the triangles are similar and the corresponding sides have the same ratio. And in general,

If the angles of one triangle are respectively equal to those of another, the triangles are similar.



7. Construct any triangle on cardboard. Then cut off the three corners and place them as in the figure. It is seen that the three angles form a straight angle. And in general,

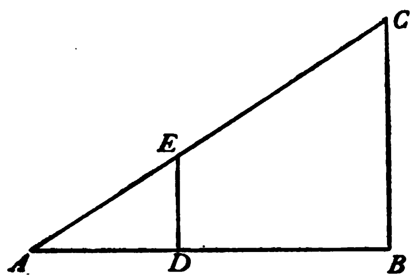
The sum of the three angles of any triangle is equal to 180 *degrees.*

8. If two angles of a triangle are 40° and 60°, respectively, what is the third angle?

9. It is thus seen that if two angles of a triangle are known, the third one can be found. Hence, the principle following problem 6 can be stated,

If two angles of any triangle are equal to two angles of another, the two triangles are similar.

10. In right triangle ABC, if a line is drawn cutting off a right triangle ADE, the triangle ADE is similar to tri-



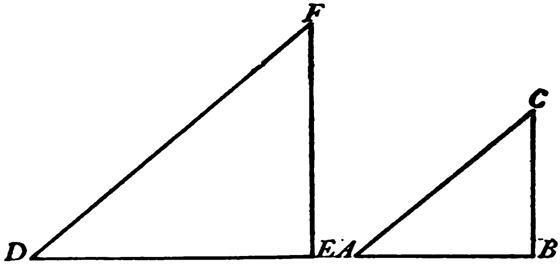
angle ABC, for they each have a right angle and angle A is common to both. If $AD = 10$ ft., $AB = 25$ ft., and $DE = 8$ ft., what is the length of CB?

2. PRACTICAL MEASUREMENT OF DISTANCES

The proportion between the corresponding sides of similar triangles gives a practical method of finding distances where direct measurement is impossible. A few such applications are shown in the following problems.

1. Since the time of the ancient Greeks, the heights of objects have been found from the lengths of their shadows.

Thus, in the figure, the height of the two objects, BC and EF; their shadows, AB and DE; and the sun's rays passing over their tops, AC and DF, form two similar triangles.



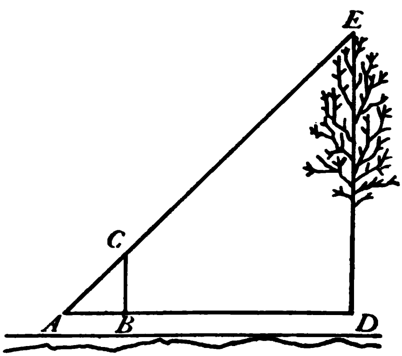
Since DE, EF, and AB can be found by measurement, BC, the unknown length, can be found from the proportion. Suppose $DE = 24$ ft., $EF = 20$ ft., and $AB = 15$ ft., by letting $BC = x$, we have the proportion

$$\frac{x}{15} = \frac{20}{24}.$$

Hence, $x = \dfrac{20 \times 15}{24}$, by multiplying both members by 15.

2. When a tree casts a shadow 48 ft. long, a vertical staff 6 ft. high casts a shadow 8 ft. long. Find the height of the tree.

3. A telephone pole casts a shadow 42 ft. long when a fence post 5 ft. high casts a shadow 6 ft. long. How high is the telephone pole?



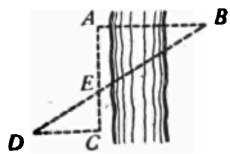
4. A boy used the following method to find the height of a tree: he used a right triangle whose sides AB and BC were equal. He found a spot in the tree, at D, just as high as his eyes. Then he walked back, sighting along AB to D, until he could see

E in line with AC. He then measured the distance AD. If AD is 60 ft., how high is the tree if D is a point 5 ft. from the ground ?

5. Using the length of shadows, find the heights of trees, telephone poles, water towers, or other heights, then check the results by use of a right triangle as in problem 4.

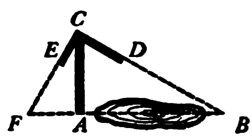
SUGGESTION. — In making the right triangle, more accurate results will be obtained if the equal sides are at least 2 or $2\frac{1}{2}$ ft. long.

6. The distance AB across a stream may be found by measuring along the shore at right angles to AB to some point C, then measuring at right angles to AC to some point D. Now sighting from D to B, mark point E where the line of sight crosses AC. AB corresponds to DC and AE to EC. Write the proportion by which AB may be found.

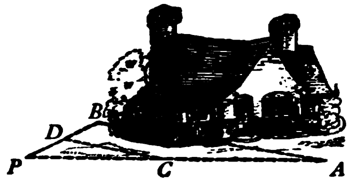


7. In the figure of problem 6, if $DC = 150$ ft., $CE = 100$ ft., and $EA = 200$ ft., find AB.

8. An easily devised instrument, as shown in the illustration, can be used to find the distance from a given point to some inaccessible point. AC is a staff at the end of which is a movable frame ECD in which EC and CD are joined at right angles. With CD pointing to the object B, and by noting a point F in the ground toward which CE points, the distance AB can be determined, for triangles ABC and ACF are similar. Find AB when $AC = 5$ ft. and $AF = 2$ ft.

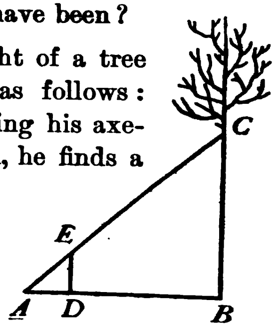


9. Find the distance from A to B by taking the following measurements: BP and AP are measured and are 240 ft. and 500 ft., respectively. Then from P, PD and PC are laid off, respectively, 24 ft. and 50 ft. (each $\frac{1}{10}$ of the BP and AB), making triangle PCD similar to triangle PAB. If $DC = 38$ ft., how far is it from A to B?

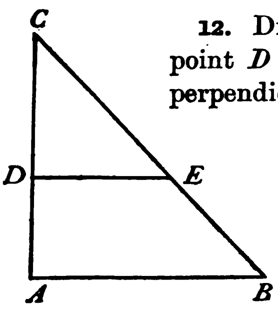


10. In problem 9, if PD and PC have been laid off equal to $\frac{1}{5}$ of BP and AP and DC had been found to be 38 ft., what would the distance from A to B have been?

11. A woodsman measures the height of a tree to the first limb very approximately as follows: Walking back from the tree and holding his axe-handle perpendicularly at arm's length, he finds a position from which the axe-handle just covers the height he wishes to find. By measuring the distance to the tree, he computes the height.



When standing 25 ft. from the tree, what is the height covered by a 30 in. axe-handle held 27 in. from his eye?

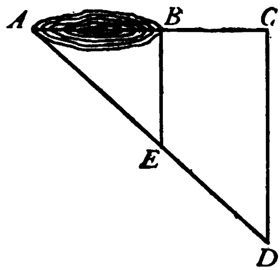


12. Draw any right triangle. From any point D draw a parallel to AB by erecting a perpendicular to AC. Carefully measure AD and DC and find the ratio of AD and DC. Then measure BE and EC and find the ratio of EB to EC.

If accurately drawn, measured, and computed, you will find the ratios to be the same. And in general,

A line parallel to one side of a triangle divides the other two sides proportionally.

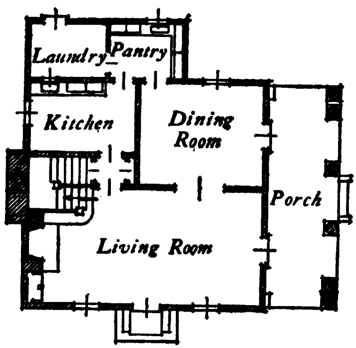
13. Wishing to measure the distance across a pond from A to B, some boys made use of the principle found in problem 12. They measured to C in line with AB. From B and C perpendiculars to AC were drawn, and points E and D on these perpendiculars, and in line with A, were marked. If $AE = 400$ ft., $ED = 300$ ft., and $BC = 200$ ft., what was AB?



14. Find heights and distances that you can measure by the principles of similar triangles.

3. MAPS AND PLANS: DRAWING TO SCALE

Maps and **plans** are figures similar to the figures which they represent. Thus, a map of a state is a drawing similar in shape to the figure formed by the state itself. The drawing in the margin represents an architect's floor plan of a house. To understand maps and plans requires a knowledge of the meaning of **drawing to scale**, for all such figures are thus drawn. The maps of any geography usually give on the map the scale to which it was drawn. Thus, a map in which 200 miles are represented by 1 inch is said to be drawn to scale 200 miles to 1 inch; or "Scale 1 in. = 200 mi."



1. Using a map in your geography and the scale to which it was drawn, find the distance in a straight line from New York to Chicago. From New York to San Francisco. From Chicago to New Orleans.

2. A map of Illinois, scale 1 in. = 200 mi., is $1\frac{5}{8}$ in. long. How long is the state?

3. On a map, scale 1 in. = 240 mi., it is $3\frac{9}{16}$ in. from Chicago to Denver. How far is it from Chicago to Denver?

4. If the floor plan shown on page 43 is drawn to scale 1 in. = 16 ft., find the dimensions of the living room; of the dining room; of the porch.

5. Draw a plan of the floor of your room to scale 1 in. = 10 ft.

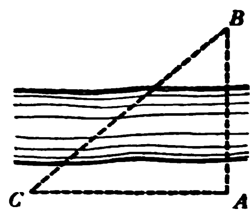
6. When the plan of a room 20 ft. by 30 ft. is 5 in. by 7.5 in., what is the scale?

7. Draw the plan of a garden 48 ft. by 120 ft. to scale 1 in. = 24 ft.

8. "Scale $\frac{1}{4}$" means that the dimensions of the plan are each $\frac{1}{4}$ of those of the thing represented. Draw to scale $\frac{1}{4}$ the plan of a rectangle 12 in. by 16 in.

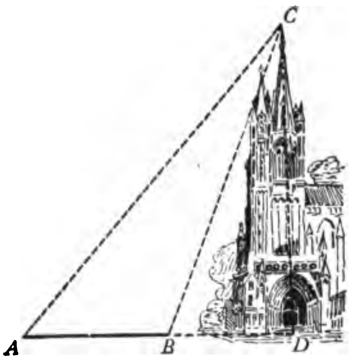
9. Draw to scale $\frac{1}{12}$ the plan of a table top 4 ft. by 7 ft.

10. The distance to an inaccessible object may be found by drawing a plan to scale. Thus, to find the distance from A to B, measure off AC perpendicular to AB. To make a map or plan, lay off any length to represent AC and from the ends of the line-segment thus taken construct angles equal to the given angles.

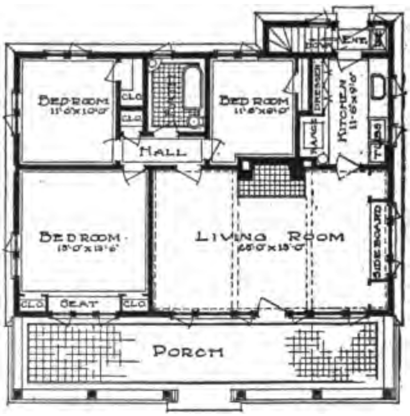


Supposing that $AC = 800$ ft., $\angle ACB = 40°$, and $\angle BAC = 90°$, construct a map to scale 1 in. = 100 ft. and compute the distance from A to B.

11. To find the height of the church spire as shown here, a line AB, 80 ft. long, toward the foot of the spire was taken. From A and B the angles of elevation of the top were taken. $\angle DAC = 50°$ and $\angle DBC = 80°$. Make a diagram to scale 1 in. = 20 ft. and find the height of the spire.



12. The following is the floor plan of a cottage. From dimensions marked, find the scale to which it was drawn. Check by using other dimensions.



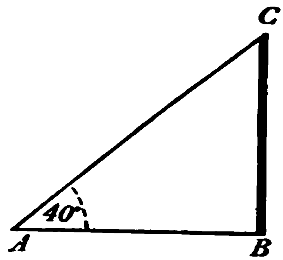
13. From the scale found, find the dimensions of the porch. Of the bathroom. Of the entire floor plan.

14. If possible, bring to class some architect's real plans for a house and interpret them.

CHAPTER VI

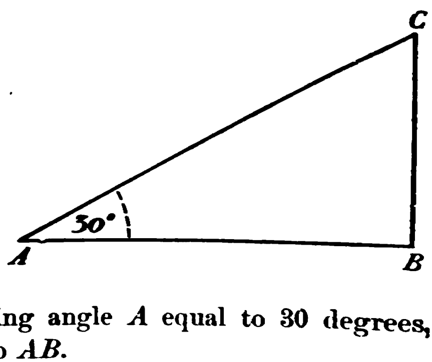
TRIGONOMETRIC RATIOS

In Chapter V you saw some of the uses of the properties of similar triangles in determining heights and distances. In this chapter a further use of ratios will be shown. It will be shown how to find the height BC of the figure represented in the margin by knowing the distance from A to B and the **angle of elevation** at A; that is, the angle formed by AB and AC.



1. TANGENT RELATIONS

1. Draw any right triangle ABC, right angled at B, with angle $A = 30°$. Measure BC and AB carefully and find the ratio of BC to AB. (The longer you can take AB the more accurate you will be likely to get the ratio.)



2. Make other triangles, using different lengths for AB, as 10 in., 15 in., or 20 in., but keeping angle A equal to 30 degrees, and find the ratio of BC to AB.

If accurately constructed and computed, you found the same ratio in each case, and to the nearest hundredth it was .58.

This follows from the fact that all the triangles you constructed were similar, and that the ratio of similar sides of similar triangles is constant; that is, the ratio is always the same.

The ratio of the perpendicular BC to the base AB is called **the tangent of angle A.** It is written $\tan \angle A = \frac{BC}{AB}$. In the problem given, it was found that $\tan 30° = .58$.

The tangent is but one of the six possible ratios, called **trigonometric ratios**, in any right triangle. Being the one used in finding heights, it is the only one defined here. The others are called **sine A, cosine A, secant A, cosecant A,** and **cotangent A.** Their meaning and use will be taken up in **Book III** of this course.

3. In the figure of problem 1, if $AB = 75$ ft., find BC.

SOLUTION

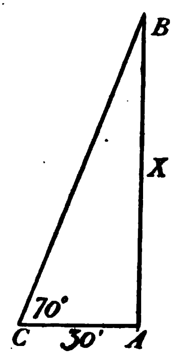
Let $BC = x$.

Then $\frac{x}{75} = .58$, for the ratio is the tangent of 30°, which is .58.

Hence, $x = 75 \times .58 = 43.5$, the number of feet.

4. In the same figure, if $AB = 120$ ft., find BC.

5. Find the height X of the flag pole AB when the angle of elevation at a point 30 ft. from the foot of the pole is 70°, having given that $\tan 70° = 2.75$.

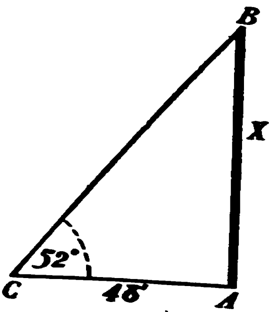


6. If the angle of elevation to the top of a church spire at a point 50 ft. from the foot of the spire is 50°, having given that $\tan 50° = 1.19$, find the height of the spire.

2. A TABLE OF TANGENTS FOR ANGLES FROM 1° TO 89°

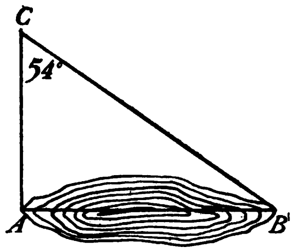
Angle	Tangent	Angle	Tangent	Angle	Tangent	Angle	Tangent	Angle	Tangent
1°	.02	19°	.34	37°	.75	55°	1.43	73°	3.27
2	.03	20	.36	38	.78	56	1.48	74	3.49
3	.05	21	.38	39	.81	57	1.54	75	3.73
4	.07	22	.40	40	.84	58	1.60	76	4.01
5	.09	23	.42	41	.87	59	1.66	77	4.33
6	.10	24	.44	42	.90	60	1.73	78	4.70
7	.12	25	.47	43	.93	61	1.80	79	5.14
8	.14	26	.49	44	.96	62	1.88	80	5.67
9	.16	27	.51	45	1.00	63	1.96	81	6.31
10	.18	28	.53	46	1.03	64	2.05	82	7.11
11	.19	29	.55	47	1.07	65	2.14	83	8.14
12	.21	30	.58	48	1.11	66	2.25	84	9.51
13	.23	31	.60	49	1.15	67	2.36	85	11.43
14	.25	32	.62	50	1.19	68	2.47	86	14.30
15	.27	33	.65	51	1.23	69	2.60	87	19.08
16	.29	34	.67	52	1.28	70	2.75	88	28.64
17	.31	35	.70	53	1.33	71	2.90	89	57.29
18	.32	36	.73	54	1.37	72	3.08		

1. The distance from an observer to the foot of a flag pole is 48 ft. The angle of elevation of the top of the pole at the point of observation is 52°. Find the height of the pole.



2. Two boys measure the height of their kite as follows: when the kite is directly over one of the boys the boy holding the kite string is 700 ft. away, and the string makes an angle of 40° with the horizontal. Find the height of the kite.

3. To find the distance AB across a small lake, AC was run at right angles to AB. AC measured 750 ft., and the angle BCA measured 54°. Find the distance from A to B.



4. A tower known to be 260 ft. high forms an angle of elevation of 25° from the point of observation. How far away is the observer?

In this problem $\dfrac{260}{x} = \tan 25° = .47$ or $\dfrac{47}{100}$.

To solve it, write it $\dfrac{x}{260} = \dfrac{100}{47}$.

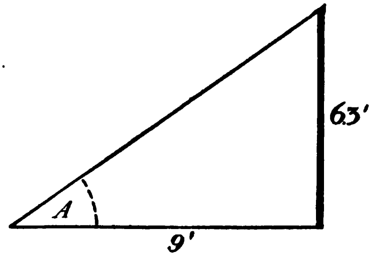
Knowing that since one acute angle is 25° the other must be 65°, we could have solved the problem from the equation:

$$\frac{x}{260} = \tan 65° = 2.14.$$

Suggestions. — Solve in both ways. The fact that the two answers do not quite agree comes from using a table true to the second decimal place only. Had $\tan 25° = .4663$ and $\tan 65° = 2.1445$ been used, the answers would have more nearly agreed.

5. A mariner finds that the angle of elevation of a light from a lighthouse known to be 80 ft. above the level of the ship, is 8°. How far away is the lighthouse? (This is like problem 4.)

6. When a vertical rod 6.3 ft. high casts a shadow 9 ft. long, what is the elevation of the sun?



SOLUTION

$$\tan A = \frac{6.3}{9} = .7,$$

In the tables, $\tan 35° = .7$.

Hence, angle $A = 35°$.

7. When a man 6 ft. tall casts a shadow 10 ft. long, what is the elevation of the sun?

8. The angle of elevation of an aëroplane from the observer was 70°. The aëroplane was directly over a point 1200 ft. away. Find the height of the aëroplane.

9. When the legs of a right triangle are 10 in. and 22.5 in., respectively, find the number of degrees in each of the two acute angles.

10. When the elevation of the sun is but 20°, how long a shadow will a boy $5\frac{1}{2}$ ft. tall cast? (See problem 4.)

11. From an aëroplane an observer notes that the angle of depression of the enemy trench is 48°, and that his elevation is 6000 ft. Find the distance from a point on level ground directly below the aëroplane to the trench.

NOTE. — The angle of depression is the same as the angle of elevation from the trench to the aëroplane.

12. From the point of observation 38 ft. above the water on a transport, the periscope of a submarine is noted at an angle of depression of 15°. How far away is the submarine?

CHAPTER VII

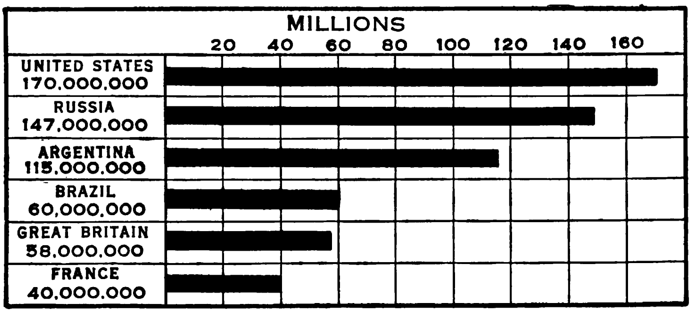
GRAPHIC METHODS OF REPRESENTING FACTS

In order to present quantitative facts so as to make the relations stand out clear and effective, writers in the newspapers and magazines, officials in making reports, advertisers, lecturers, and others attempt to picture the relations to the eye. These pictured forms of presenting facts are called **graphic methods of presentation.**

The graphic method has not become a fixed form of presentation yet, as forms of computation have. The graphs are nearly as varied in form as the number of persons using them. They are roughly classified, however, under three general heads: **straight lines or bars; broken lines;** and **circles.**

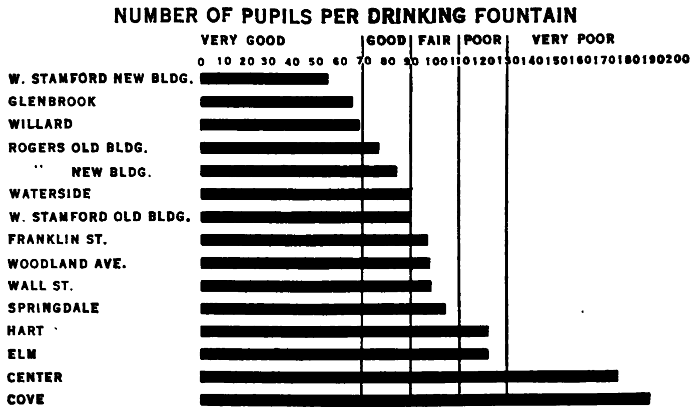
The following graph is one of the best forms of the bar graph.

THE NUMBER OF MEAT-PRODUCING ANIMALS IN 1915. THESE INCLUDE CATTLE, SHEEP, AND PIGS.



1. GENERAL ILLUSTRATIONS

1. The following graph is taken from a school report of the Stamford, Connecticut, schools, showing the number of pupils per drinking fountain. It is a splendid graph for this purpose. Thus, it is seen at a glance that the conditions at three of the schools are "very good." Of these three, the conditions in the West Stamford Building are the best, having but about 55 pupils per fountain.



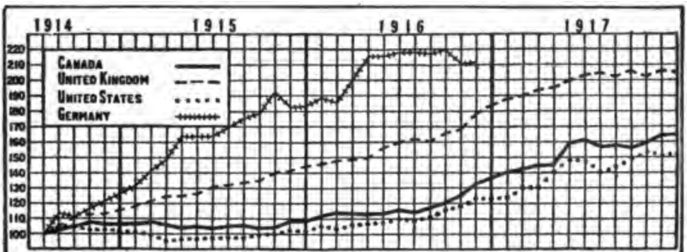
2. From the above chart, tell approximately the relation of the number per drinking fountain in the Hart and Elm Schools when compared with the number in the West Stamford Building.

3. From the chart, tell approximately how many per cent more per fountain in the Hart and Elm Schools than in Woodland Avenue School.

SUGGESTION. — Since the Woodland Avenue School is about 100, and the Hart School about 120, it is 20 % more.

4. The percentage is about how much less in the Willard School than in the Franklin Street School?

5. The following graph, taken from *The Literary Digest* of March 16, 1918, showing how food prices increased during the War, is a typical chart for picturing changing relations. It could have been improved by placing the numbers at the right of the chart as well as at the left. The numbers do not indicate any particular price, but show what per cent they were of the price at the beginning of 1914.



HOW PRICES HAVE RISEN IN NATIONS AT WAR.

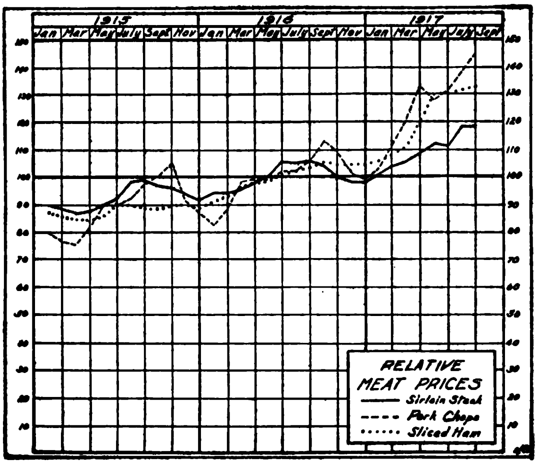
6. Observing that the graph starts at 100, 120 shows a 20 % increase; 150 shows a 50 % increase; 180, an 80 % increase. About what was the per cent of general increase in Canada at the end of 1915? At the end of 1916? At the end of 1917?

7. What was the per cent of general increase in Germany at the end of 1915? At the middle of 1916?

8. What was the per cent of general increase in the United States at the end of 1915? At the end of 1916? At the end of 1917?

9. What was the per cent of increase in England at the end of 1915? At the end of 1916? At the end of 1917?

10. The following graph is taken from an article on "The Trend in Food Prices" by R. T. Bye, published in *The Annals of the American Academy of Political and Social Science*, November, 1917. The heavy line marked 100 is the average price for 1916. How much lower was sirloin steak in March, 1915, than the average price of 1916? Sliced ham? Pork chops?



11. Compare the price of sliced ham in May, 1915, with the price in July, 1917.

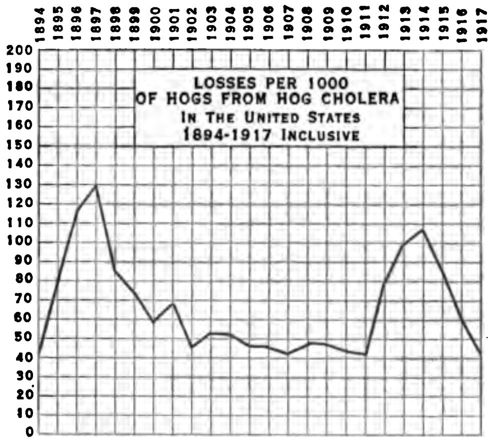
12. Compare the price of pork chops in March, 1915, with the price in July, 1917.

13. Compare the price of sirloin steak in July, 1916, with the average price of 1916.

14. Compare the price of sirloin steak in July, 1917, with the average price of 1916

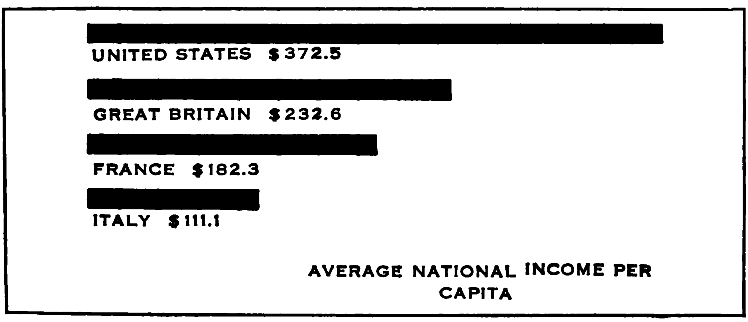
15. Make other interesting comparisons as your teacher may direct.

16. The following graph, taken from *The Scientific Monthly*, May, 1918, shows the variation in losses from hog cholera from 1894 to 1917, inclusive.

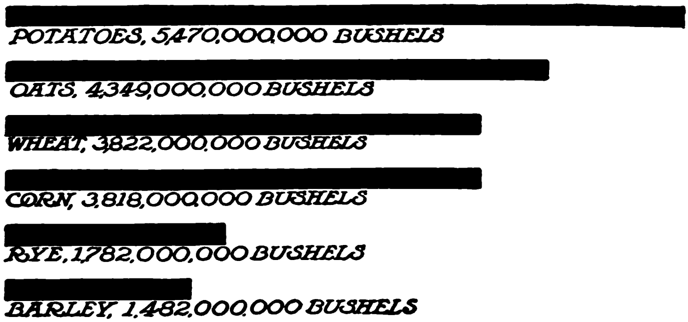


Does the graph indicate a control of the disease?

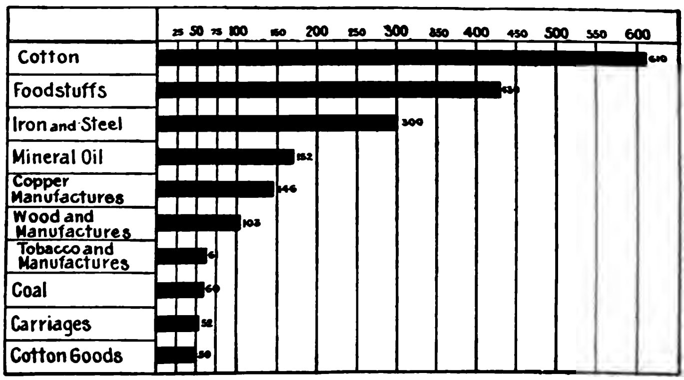
17. During the drive for the Fourth Liberty Loan in the fall of 1918, *The New York Times* published the following graph showing the relation of our national income per capita to that of our allies. Compare our per capita income with that of the other three countries here represented.



18. The following chart, taken from *The Country Gentleman* of June 9, 1917, shows the world's production of six leading crops. From the chart, the production of rye is what per cent of the production of corn? Compare the production of potatoes with that of wheat.



19. The following graph of our exports, on account of the perpendicular lines and the data written both at the top and at the right of the bars, is a better type for showing data, for the comparisons can be more easily made. It is taken from *The World's Work* of December, 1914.



Compare the cotton exports with the foodstuffs. Foodstuffs with iron and steel. Mineral oil with iron and steel.

20. The exports of coal were about what per cent of those of iron and steel?

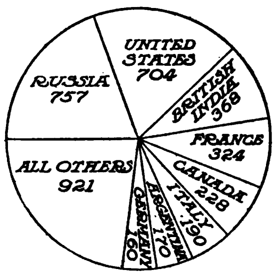
21. The iron and steel exports were about what per cent of the cotton exports?

22. By the use of your ruler or with compasses, compare the exports of cotton and foodstuffs with all the rest combined.

23. Make other comparisons as your teacher may direct.

24. Formerly, the circle was much used to show the relation of parts to the whole. Such relations are now more often shown by a bar, shaded to represent different parts. The following graph, taken from *The Country Gentleman* of June 9, 1917, shows a type of circular graphs.

CORN



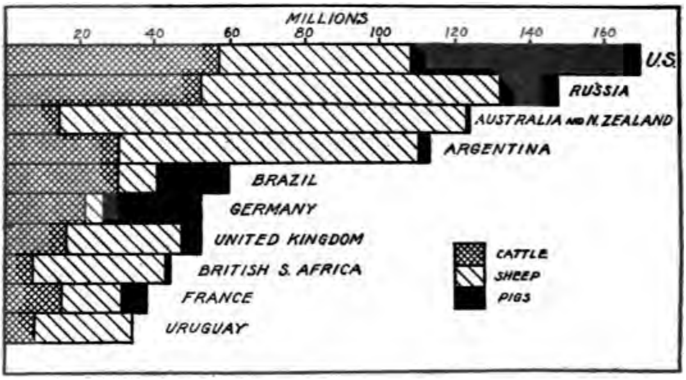
WHEAT

25. About what per cent of the world's production of corn does the United States produce?

26. About what per cent of the world's wheat crop is produced by the United States? By Russia? By France?

27. From the graph, rank in order the five leading wheat-producing countries.

28. The bar is coming to be more used than the circle in representing the component parts of the whole. It is more easily read and more easily made. Thus, a graph like the following by G. B. Roorbach in *The Annals of the American Academy of Political and Social Science*, November, 1917, is more often used in careful discussions. How is the number of cattle, sheep, and hogs distributed in the United States?



29. Which country has the greatest per cent of sheep? The smallest per cent?

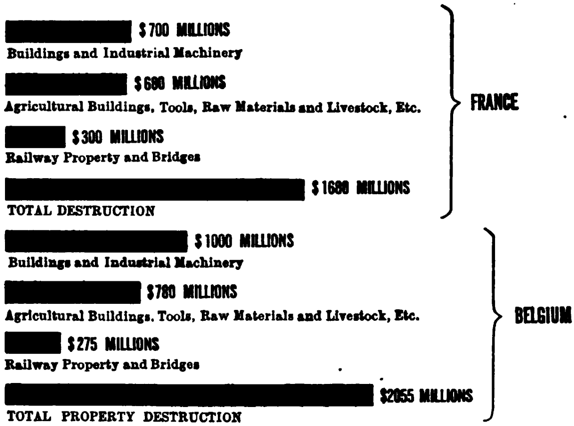
30. Which country produces the greatest per cent of hogs? Which one, except Uruguay, the least?

31. About what per cent of our food animals are cattle? Sheep? Hogs?

32. What per cent of Uruguay's animals are cattle? Sheep?

33. Show in per cents the distribution of hogs, sheep, and cattle in France.

34. The graph shown here was published in newspapers and magazines during the War to show the property destroyed by Germany in France and Belgium during the first two years of the War.



a. Using a pair of compasses, tell what per cent of the total destruction in France was buildings and industrial machinery; tools, raw material, live stock, etc.; railroad property.

b. In the same way, make similar comparison of the destruction in Belgium.

c. Compare the total destruction of property in Belgium with that of France.

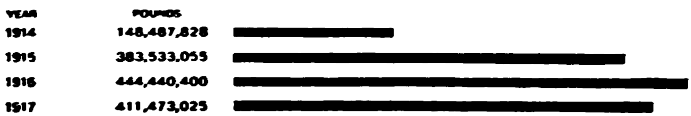
d. Compare Belgium's loss of buildings and industrial machinery with that of France.

35. Make a collection of graphs from newspapers, magazines, reports, and other sources. Paste them upon cardboard and leave with your teacher to illustrate the various types. Study them carefully as to whether or not they present accurately and definitely the quantitative relations.

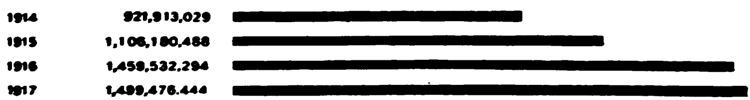
2. SIMPLE COMPARISONS

In the following problems, use the horizontal bar to show comparisons. Place the actual figures from which the graphs were constructed at the left end of the bars. The following from Swift & Company's Year Book of 1918 is a good example of a very common type of the horizontal bar graph.

Beef Products



Pork Products



The comparison could have been more easily made had perpendicular lines divided each of these bars into sections representing 25,000 pounds each, as in problem 19, page 56.

1. From the graph shown here, compare the exports of beef produced in 1915, 1916, and 1917 with those of 1914. That is, tell about how many times as great. Use a pair of compasses in making the comparisons.

2. Compare the same exports by telling how many per cent more one is than the other.

3. From the data, compute the per cents and see how nearly they agree with your estimates from the graphs.

4. It was estimated that the per capita consumption of sugar in 1915 in the following six countries was: France, 39.01 lb.; England, 89.69 lb.; United States, 83.83 lb.; Russia, 29.26 lb.; Italy, 10.45 lb.; and Belgium, 42.79 lb.

Make a horizontal bar graph showing comparisons.

If you represent 20 lb. by 1 in., how long will the bar be for France? For England? For the United States?

5. Show by graphs the relative importance of beans as a food crop, from the productions of the following countries before the War: India, 125 million bushels; Italy, 23 million bushels; Japan, 21 million bushels; Austria-Hungary, 19 million bushels; Russia, 12 million bushels; and the United States, $11\frac{1}{2}$ million bushels.

SUGGESTION. — Select some length to represent a given number of bushels. Thus, if you select 1 inch to represent 25 million bushels, then compute what length must be used for each.

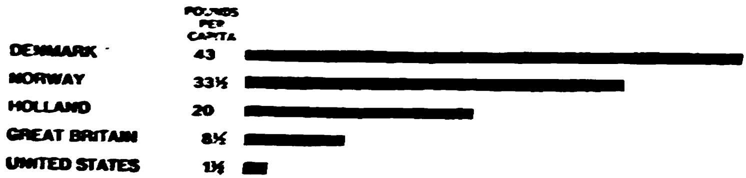
6. Always try to make your graph show clearly the relations that you are trying to picture. Thus, to picture the following, draw perpendicular lines 1 inch apart, and let each 1-inch space represent 2 bushels, and see if your graphs are more easily interpreted than the others you have drawn.

The per capita consumption of potatoes in four countries is: United States, 2.6 bu.; France, 7.7 bu.; England, 8.3 bu.; Belgium, 9.4 bu.

SUGGESTION. — Use a graph similar to that on page 51.

7. Show graphically the relations of our four Liberty Loan Bond sales of 1917 and 1918. They were: First Loan, \$3,035,226,850; Second Loan, \$4,617,532,300; Third Loan, \$4,176,516,850; Fourth Loan, \$6,989,047,000.

8. The following graph shows the per capita consumption of oleomargarine for various countries, as given in *The Independent* of November 17, 1917. Draw a graph of the same, ruling perpendicular lines 1 inch apart and let each space represent 10 lb.



9. The per capita consumption of meat by various countries was approximately as follows: in 1902, Australia, 263 lb.; in 1902, New Zealand, 212 lb.; in 1909, United States, 171 lb.; in 1910, Canada, 137 lb.; in 1906, Great Britain, 119 lb.; in 1904, France, 79 lb.; in 1902, Belgium, 70 lb.; in 1899, Russia, 50 lb. At the left of the bars give both the name of the country and the year. Draw perpendicular bars about ¼ in. apart to represent 10 lb. Mark these at the top of the graph, 0, 10, 20, 30, etc.

Try other plans of graphing the same data and see which you prefer. For example, draw such a graph as the one shown in problem 8, letting 50 lb., 60 lb., or any convenient number be represented by 1 in.

10. The average yearly production of wheat for three years (1911–1913) for several countries, given in million bushels, was as follows: Russia, 727; United States, 705; Italy, 191; Australia, 89; Great Britain, 61; Belgium, 15. Show the relations by graphs.

11. In 1913 the production of cane sugar in short tons (2000 lb.) was as follows for the seven countries leading in the production: Cuba, 2,909,000; India, 2,534,000; Java,

1,591,000; Hawaii, 612,000; Porto Rico, 398,000; Australia, 397,000; United States, 300,000. Show the comparison by graphs.

12. The following is the per cent of games won by the various clubs of the National League in 1918. Chicago, 65.1 %; New York, 57.3 %; Cincinnati, 53.1 %; Pittsburg, 52 %; Brooklyn, 45.2 %; Philadelphia, 44.7 %; Boston, 42.7 %; St. Louis, 39.5 %. Show the relations graphically.

13. Our production of grain in 1918 was as follows: wheat, 918,920,000 bu.; corn, 2,717,775,000 bu.; oats, 1,535,297,000 bu.; barley, 236,505,000 bu.; and rye, 76,687,000 bu. Show the relations graphically.

14. The price of hogs, live weight, per 100 lb. varied as follows during a five year period: in 1914, $7.45; in 1915, $6.57; in 1916, $6.32; in 1917, $9.16; in 1918, $15.26. Taking the years in order, show graphically the variation in price.

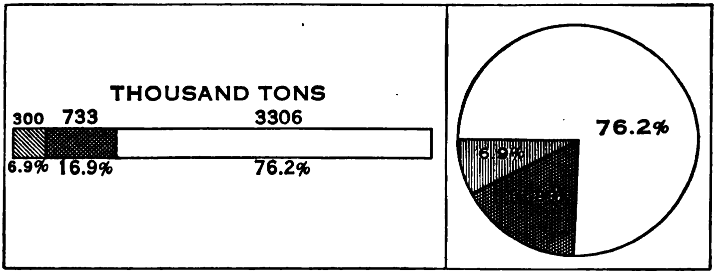
15. The average price per acre of cultivated land in 1918 in six states was as follows: Maine, $35; Massachusetts, $68; Illinois, $139; Indiana, $106; Iowa, $156; California, $110. Arrange in regular order from highest to lowest and represent graphically.

16. Take any data your teacher may direct and draw neat graphs, showing comparisons, until you are able to make and interpret graphs easily.

3. GRAPHS SHOWING COMPONENT PARTS

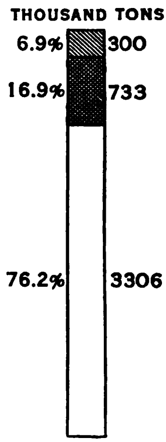
The relation of a part to the whole is usually shown either by the circular graph or by a shaded bar graph. Of the two methods, the bar graph is more easily made and more

easily read. Thus, the relation of the production to the importation of sugar in the United States yearly may be shown by either of the following methods:



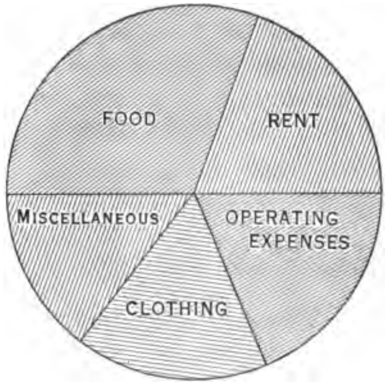
The 6.9 % is the cane sugar; the 16.9 %, is the beet sugar; and the 76.2 % is the importations.

The bars are often made perpendicular as is shown on the margin. It makes but little difference and depends largely upon what statements the maker of the graph wishes to record in the graphs and the ease with which it may be read.

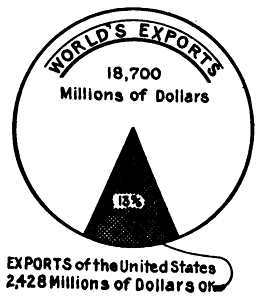


1. To make a circular graph, divide the given whole by 360 to find what each degree must represent. Thus, in the graph shown above, 360° must represent 4339 thousand. Hence, each degree must represent about 12.1 thousand. Hence, to represent 300 thousand will require as many degrees as 300 will contain 12.1, or 24.8. In like manner, find how many degrees must represent 733 thousand and 3306 thousand. Before drawing the graph, see if the sum of all three arcs is 360°. Why? Now with a protractor draw a graph like the one shown here.

2. While no one making a graph to show more clearly and vividly a set of facts will fail to place the data from which it was made where they can be clearly seen and read, let us suppose that such a graph as the one in the margin should be made to represent the disposition of a family income of $1500. By use of your protractor, find the approximate amount allowed for each item.



3. In the graph given here, taken from *The World's Work*, December, 1914, the 13 % furnishes sufficient data from which to find the amounts. Compute, both by per cent and by measuring the arc with a protractor, the exports of the United States, and see if the results agree with the figures given here.



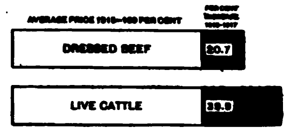
You will find that neither method gives *exactly* 2428 million. But the graph pictures to the eye *approximate* relations. The graph, then, is accurate enough to convey the facts to the general reader.

4. In 1917 we consumed about 180 pounds of meat per capita, distributed as follows: pork, 92 pounds; beef, 82 pounds; mutton, 6 pounds. Draw a circular graph showing the relation of each to the whole.

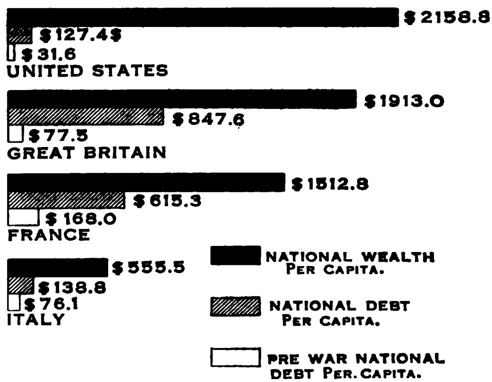
SUGGESTION. — How many degrees must be taken to represent 1 pound? How many to represent 92 pounds? 82 pounds? 6 pounds?

(Check your answers before drawing the graph; that is, see if the sum is 360°.)

5. Swift & Company's Year Book of 1918 uses the following bar graph to show the relative increase in the price of cattle and of dressed beef from 1915 to 1917, based upon data from *The Chicago Daily Drovers' Journal.* The graph shows what the relation is between the gains in prices of cattle and of dressed beef. The figures show what relations?



6. The following graph was published in *The New York Times*, October 13, 1918, in promoting the Fourth Liberty Loan.



a. Compare our national debt in 1918 with our national wealth.

b. Compare our national debt in 1918 with our prewar national debt.

c. Compare our national debt per capita with that of our three allies.

d. Compare our national wealth per capita with that of our three allies.

e. Compare Great Britain's national debt with her national wealth.

f. Compare the national debt of France with her national wealth.

g. Compare Italy's debt with her wealth.

7. Show by a bar graph the relations shown by the circular graph in problem 4.

8. We produce yearly an average of 705 million bushels of wheat, and export 116 million bushels of it. Show the relation by a bar graph. Show the same by a circular graph. Which method of presentation do you prefer? Why?

9. Great Britain consumes 282 million bushels of wheat yearly and has to import 221 million bushels of it. Show the relation by a bar graph.

10. France raises normally 324 million bushels of wheat and exports 55 million bushels of it. Show this by a perpendicular bar graph.

11. Cuba produces 2,909,000 tons of sugar yearly and exports 2,738,000 tons of it. Show the relation by any type of graph you wish.

12. In a recent year 86 % of the world's total export of meat was supplied by five countries: Argentina and Uruguay together, 36 %; United States, 31 %; and Australia and New Zealand together 19 %. Show by a bar graph the relation of each to the whole.

13. Make a bar graph showing the relation of the boys and the girls to the entire enrollment of your room. Of the entire school.

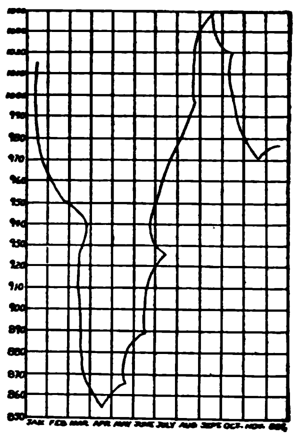
14. Make a bar graph showing the relation of boys and of girls to the entire number studying second-year Junior High School Mathematics.

15. By circular graphs show the same relations shown by the graphs of problems 13 and 14.

4. CURVE PLOTTING: THE BROKEN LINE GRAPH

Information is shown graphically in many different ways. The method used depends largely upon the user, and not upon the facts presented. Yet, in a general way, the broken line, sometimes called "curve plotting," is more often used than other methods to show the variation through which quantities pass. Thus, to show the variation in the price of a commodity over a fixed period, a **broken line graph** is more often used than other forms of graphs.

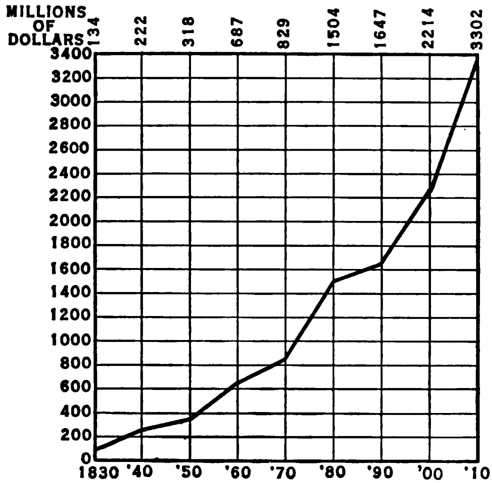
In using the broken line or "curve," the zero line should be shown or the chart should show an irregular line at the bottom, thus showing "an incomplete chart." Thus, from a first glance at the chart in the margin, taken from *The Country Gentleman* of January 6, 1918, the impression is that calves sell for about half as much in March and in June as in January and September. But the curve begins at $8.50, and so, when properly interpreted, the price has not made such a change as a chance observation would lead one to conclude.



THE SELLING PRICE OF CALVES IN 1917.

1. By use of a zero line, as in the figure on page 69, one does not draw the wrong conclusion that would be possible with the graph shown above. Thus, one sees at a glance that the value of the exports of 1870 was but about one third of those of 1900, or that our exports in 1900 had nearly doubled since 1870. Compare the exports of 1880 with those of 1850. Those of 1890 with those of 1840.

Those of 1880 with those of 1910. This graph is one of the most approved types of curve graphs.



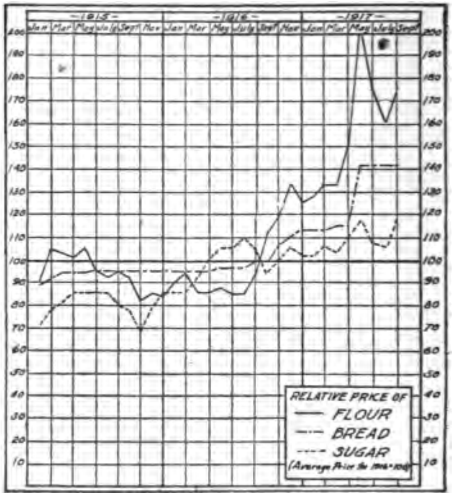
THE VALUE OF OUR EXPORTS FROM 1830 TO 1910.

2. Show the relations given in the graph of problem 1 by horizontal bar graphs, placing the year and the exports at the left end of each bar.

3. The average prices received by farmers for wheat during the first ten months of 1917 were: Jan., \$1.50; Feb., \$1.65; Mar., \$1.64; Apr., \$1.80; May, \$2.46; June, \$2.49; July, \$2.20; Aug., \$2.29; Sept., \$2.10; Oct., \$2.00. Show by a curve graph the variations in price.

4. The average September price received by the farmer for hogs, per 100 lb. live weight, ranged for eight years as follows: 1910, \$8.27; 1911, \$6.53; 1912, \$7.47; 1913, \$7.68; 1914, \$8.11; 1915, \$6.79; 1916, \$9.22; 1917, \$15.69. Show the variations by a curve graph. Show them by horizontal bar graphs.

5. The following graph is taken from *The Annals of the American Academy of Political and Social Science* of November, 1917.



Study the graph and answer the following questions and similar ones that your teacher may ask.

NOTE. — The figures do not show any particular price, but show what per cent they were of the average price of 1916, shown by the heavy line marked 100.

a. In May, 1917, the price of flour had increased how much over the average price of 1916?

b. It was how much less in September, 1915, than the average price of 1916?

c. Sugar was how much higher in May, 1917, than at the end of March, 1916?

d. Sugar was how much lower at the end of September, 1915, than the average price of 1916?

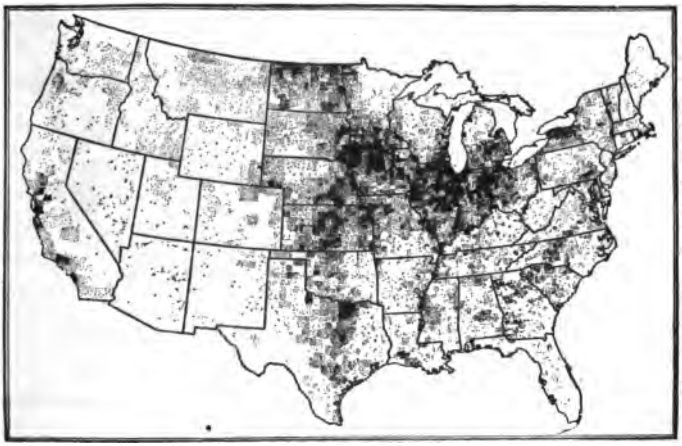
e. Sugar was how much higher the middle of July, 1916, than the average price for the year?

6. From the reports of the Board of Education of your city, make a chart showing the variation in the total enrollment in your schools for a period of years.

7. Show by a similar chart the variation of enrollment of sixth grade pupils for a number of years.

5. MAP PRESENTATION OF FACTS

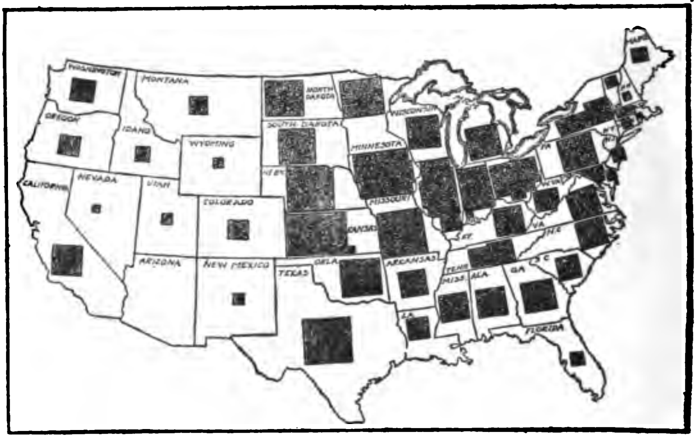
Maps marked or shaded in various ways form a very convenient way of presenting information. The following is a very common form.



EACH DOT REPRESENTS A FARM TRACTOR IN USE.

1. What general section of the country uses more farm tractors?
2. What state uses the least number per acre?
3. What part of New York state uses the greater number?
4. In what general section is the use distributed more equally?
5. Compare the use in Indiana and Illinois.
6. Compare the number used in Wisconsin with the number in Missouri.
7. In what five states is the greatest use made of farm tractors?

8. The following map, taken from *The Literary Digest* of May 18, 1918, shows area in which food might be grown. The shaded areas show the amount of cultivated land in each state.



Name four states in which most of the available land is under cultivation.

9. In what state is the smallest proportion of the land under cultivation?

10. About what per cent of Washington is under cultivation?

11. About what per cent of Minnesota is under cultivation?

12. About what per cent of the New England states is under cultivation? About what per cent of the Southern states?

13. Which has the greater per cent of its area under cultivation, North Dakota or South Dakota?

14. Compare the area under cultivation in Nevada with that in Utah.

15. Compare the area under cultivation in Montana with that in Colorado. With that in Oregon.

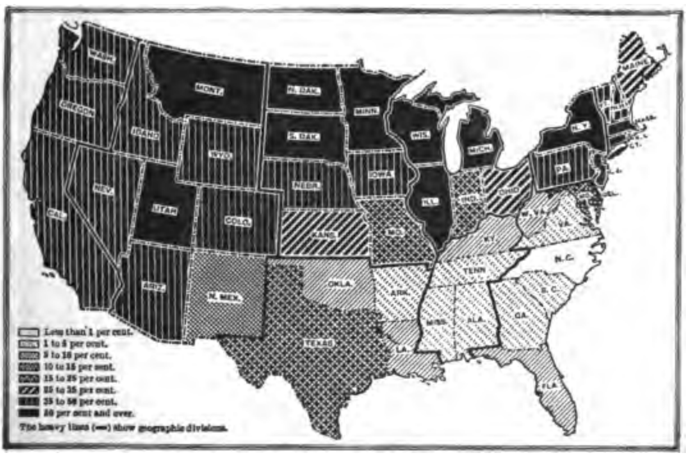
16. Compare the cultivated land in Oklahoma with that of Texas.

17. Compare the cultivated land in Georgia with that of each of the other Southern states.

18. Compare the cultivated land of Pennsylvania with that of Ohio. With that of California.

19. Try to find other maps of this nature in newspapers and magazines, and bring to class for such problems as those given here.

20. The following map, reproduced by special permission from *The National Geographic Magazine*, February, 1917, shows the foreign stock in the population of the United States. That is, the foreign-born and the children of at least one foreign-born parent.



THE FOREIGN STOCK IN OUR POPULATION

21. Name the states having a population of less than half of native stock.

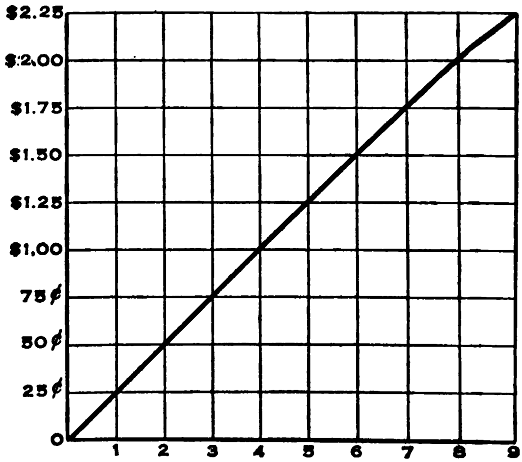
22. Name the states whose population is made up of more than one-third of foreign stock.

23. What state has more than 99 out of every 100 of native stock?

24. What general section of the country has the largest per cent of native stock? What section the least per cent?

6. FUNCTIONAL RELATIONS SHOWN BY GRAPHS

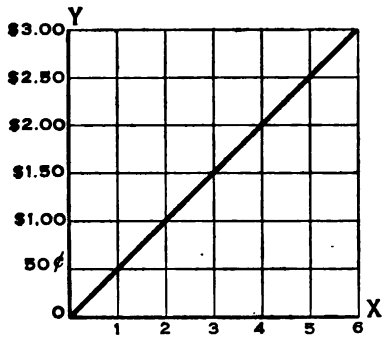
When one number depends upon another, as when the cost depends upon the amount purchased, one is said to be a **function** of the other. Thus, the amount bought is a function of the cost; the dimensions of a rectangle are functions of its area; the diameter of a circle is a function of its circumference. The graph in the margin is a **price graph** of gasoline when the price is 25 ¢ per gallon. Since the cost of 1 gal. is 25 ¢, we make a dot above 1 and opposite 25 ¢; then above 2 and opposite 50 ¢; etc.



By finding several such points and connecting them, the

graph is seen to be a straight line. Hence, but two points need to be found in order to determine such a graph.

In a graph of this kind, two principal lines at right angles are used to represent the numbers. These are called the **axes** of the graph. The horizontal one is called the ***x*-axis** and the perpendicular one the ***y*-axis**. These axes, however, are often given special names, as **axis of gallons** and **axis of cost**.



By properly numbering the axes, the diagonal of a square may always be used in a **price graph**. Thus, in the graph in the margin, when articles are selling at 50¢ each, numbering the division on the *y*-**axis** 50, 100, 150, 200, etc., and those on the *x*-**axis** 1, 2, 3, 4, etc., the cost of any number of articles may be found.

1. Draw a price graph of gasoline at 26¢. From it give the cost of 5 gal.; 10 gal.; 8 gal.

2. Draw a price graph of cloth at 45¢ per yard. From it give the cost of 5 yd. Of 7 yd. Of 12 yd.

3. Draw a price graph of potatoes at $2 per bushel. From it give the cost of 4 bu. Of 8 bu. Of 12 bu.

4. Draw a wage graph of wages at 40¢ per hour. From it give the wages for 8 hr. For 12 hr. For 9 hr.

5. Draw an interest graph showing the interest at 6%. From it give the interest of $800. Of $500. Of $1200. Of $1500.

6. Where the employer is paying several rates for help, as 25 ¢, 30¢, 40 ¢, and 45 ¢ per hour, the graphs could all be shown upon the same chart as follows: Since the wages for no time is nothing, all graphs start with zero. But one more point is needed for each graph. For example, at 25 ¢, the wages for 8 hr. are $2.00. Hence, mark a point above 8 and opposite $2.00 and draw a straight line through this point and zero. To make the graph for the 30 ¢ wage, find the wage, say for 5 hr. This is $1.50. Hence, draw a straight line through the point directly above 5 and opposite $1.50. Show how to draw the other two. From the graph, give the wages for 6 hours at each rate.

7. Make a wage graph to show the wages at 50 ¢, 60 ¢, and 75¢ per hour. From it, give the wages at each rate for 5 hr. For 6 hr. For 8 hr.

8. Let every other division on the x-axis be called 1, 2, 3, etc., so as to read wages for half hours, and make a wage graph at 28 ¢ per hour. Give the wages for $4\frac{1}{2}$ hr. For $7\frac{1}{2}$ hr.

9. Draw a price graph from which the cost to $\frac{1}{4}$ of a pound may be read when the price is 32¢ per pound.

10. Draw a graph from which the cost of butter can be read from the cost of 1 oz. to the cost of 4 lb., when butter is 48 ¢ per pound.

CHAPTER VIII

MEASUREMENTS, CONSTRUCTIONS, AND OBSERVATIONS

THIS chapter is a review of various problems in measurements that you have had in the other grades and an extension of the subject to include new areas and volumes, as well as a new process called **square root.**

1. MEASURING ANY QUANTITY: DENOMINATE NUMBERS

The **numerical measure** of any quantity is the number of times it will contain some **standard unit of measure.**

A denominate number is a number of standard units of measure, as 5 feet, 6 pounds, 8 bushels, etc. When a number consists of two or more related units, as 3 bu. 2 pk.; 5 ft. 8 in.; 3 gal. 2 qt.; etc., it is called a **compound denominate number.**

Compound denominate numbers are changed to single units by the laws of arithmetic which you already know.

1. Reduce 8 ft. 9 in. to inches.

SOLUTION

8 ft. = 8 × 12 in. = 96 in.
96 in. + 9 in. = 105 in.

The work may be shortened by using abstract numbers and computing as in the right-hand margin.

$$\begin{array}{r} 8 - 9 \\ \underline{12} \\ 96 \\ \underline{9} \\ 105 \end{array}$$

2. How many quarts in 16 gal. 3 qt.?

3. How many pecks in 64 bu. 3 pk.?

4. How many feet in 18 yd. 2 ft.?

5. How many minutes in 15 hr. 48 min.?
6. How many seconds in 38 min. 16 sec.?
7. Reduce 12 qt. 1 pt. to pints.

Reduce:

8. 12 A. 96 sq. rd. to square rods.
9. 14 mi. 96 rd. to rods.
10. 5 mi. 1230 yd. to yards.
11. 16 lb. 11 oz. to ounces.
12. 5 sq. ft. 84 sq. in. to square inches.
13. 5 cu. yd. 16 cu. ft. to cubic feet.
14. 284 in. to feet and inches.

SOLUTION

284 in. ÷ 12 in. = 23 times and 8 in. undivided.
Hence, 284 in. = 23 ft. 8 in.

EXPLANATION. — The quotient shows the relation of the dividend to the divisor. Hence, the 23 and a remainder of 8 in. shows that the dividend is 8 in. more than 23 times the divisor (12 in.). But 12 in. is a foot. Hence, the quotient shows that 284 in. is 23 ft. and 8 in.

15. Change 18 qt. to gallons and quarts.

Reduce:

16. 125 pt. to quarts and pints.
17. 196 in. to feet and inches.
18. 340 in. to yards and inches.
19. 426 sq. rd. to acres and square rods.
20. 324 oz. to pounds and ounces.
21. 175 pk. to bushels and pecks.
22. 185 ft. to yards and feet.
23. 342 min. to hours and minutes.

24. Reduce $\frac{7}{8}$ bu. to lower units.

SOLUTION

$\frac{7}{8}$ bu. = $\frac{7}{8} \times 4$ pk. = $3\frac{1}{2}$ pk.
$\frac{1}{2}$ pk. = $\frac{1}{2} \times 8$ qt. = 4 qt.
Hence, $\frac{7}{8}$ bu. = 3 pk. 4 qt.

NOTE. — This does not differ from the reduction of a whole number to a lower unit.

Reduce to lower units:

25. $\frac{3}{4}$ ft.	**29.** $\frac{7}{8}$ lb.	**33.** $\frac{4}{15}$ hr.	**37.** $\frac{5}{6}$ ft.
26. $\frac{2}{3}$ yd.	**30.** $\frac{23}{32}$ A.	**34.** $\frac{5}{8}$ A.	**38.** $\frac{3}{4}$ lb.
27. $\frac{7}{8}$ gal.	**31.** $\frac{15}{16}$ bu.	**35.** $\frac{5}{16}$ T.	**39.** $\frac{3}{4}$ gal.
28. $\frac{5}{18}$ yd.	**32.** $\frac{5}{9}$ yd.	**36.** $\frac{14}{15}$ min.	**40.** $\frac{7}{8}$ gal.

41. What part of an hour is 32 min. 40 sec.?

SOLUTION

1 hr. = 60×60 sec. = 3600 sec.
32 min. 40 sec. = 32×60 sec. + 40 sec. = 1960 sec.
1960 sec. ÷ 3600 sec. = $\frac{1960}{3600} = \frac{49}{90}$ or .544+.

42. What part of an hour is 12 min. 30 sec.?

43. What part of a yard is 2 ft. 4 in.?

44. What part of a gallon is 3 qt. 1 pt.?

45. What part of a mile is 960 ft.?

46. What part of a mile is 720 yd.?

47. Reduce 2 pk. 6 qt. to a decimal part of a bushel.

48. Reduce 9 hr. 48 min. to a decimal part of a day.

49. Reduce 1 ft. 10 in. to a decimal part of a yard.

50. What decimal part of a square foot is 96 sq. in.?

51. If a coat rack is to contain 8 hooks placed at equal distances apart, find the distance if it is 63 inches from one end hook to the other.

SUGGESTION. — By use of a diagram, show that 63 inches is to be divided into 7 equal spaces, and hence, that the divisor is 7 instead of 8.

52. If 12 plants are to be set at equal distances apart, in a row measuring 16 ft. 6 in. from the first plant to the last, how far apart must they be set?

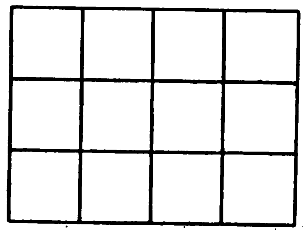
53. If posts for a fence are set 10 ft. apart, how many will be needed for a fence row measuring 210 ft. from one end post to the other?

54. How many posts set 8 feet apart are needed for a grape arbor 56 feet long? How many crosspieces will be needed if there are 3 between each pair of posts?

2. A REVIEW OF AREAS

The **area** of a surface is the measure of it when the unit of measure is the surface of some square whose side is some linear unit.

1. Show from this figure that the number of square units in the surface of any rectangle is the product of the number of linear units in its length and in its width. To do this, show: (1) that a strip across the length and 1 unit wide contains as many square units as there are linear units in the length; (2) that there are as many such strips each 1 unit wide as there are linear units in the width; and (3) that the area of one strip multiplied by the number of strips gives the entire area.



2. Let A represent the number of units in the area of a rectangle l units long and w units wide, and write the formula showing the relations.

3. From the formula $A = lw$, what does l equal in terms of A and w? w in terms of A and l?

4. Express in words the meaning of the formula, $l = \frac{A}{w}$. Of $w = \frac{A}{l}$.

5. What will a sidewalk 4 ft. wide and 120 ft. long cost at 18 ¢ per square foot?

6. What will the linoleum cost for a kitchen floor 12 ft. by 15 ft. at $1.75 per square yard?

7. How many square feet of sodding are required for a lot 80 ft. by 220 ft., deducting for a building 34 ft. by 36 ft., a walk 4 ft. by 24 ft., and 350 sq. ft. for shrubbery beds?

8. If a garden plot 42 ft. by 76 ft. is surrounded by a sod border 4 ft. wide, how much is left for cultivation? How many square feet in the sod border?

9. When a boy has mowed a strip 10 ft. wide about a rectangular lawn 70 ft. by 185 ft., what per cent of the lawn has he yet to mow?

10. Find the cost of building a sidewalk in your town or city, and compute the cost of five pieces of sidewalk in the neighborhood, in order to get an idea of the cost of any piece of sidewalk that you see.

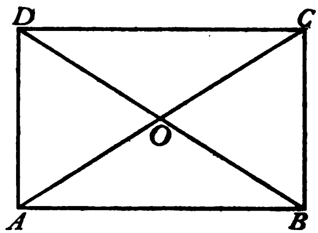
11. Find the cost of lathing and plastering a room, then find the cost to plaster a room the size of your schoolroom, and thus get an idea of the cost of plastering any surface you see.

12. Find the cost of paving streets, and compute the cost of paving a piece of street one block long in your neighborhood, and thus be able to have some notion of the cost of other pieces of paving that you see.

13. Find the cost of flooring, the amount to be added to the surface covered to allow for "tongue and groove," and compute the cost of flooring a room the size of your schoolroom, and thus be able to estimate the cost of flooring other rooms.

3. CONSTRUCTIONS AND OBSERVATIONS

By carefully constructing figures and measuring certain parts, many useful facts may be discovered. This phase of mathematics is sometimes called **constructive** or **observational geometry**.



1. Carefully construct three or more rectangles of different dimensions. By the use of a pair of compasses compare the diagonals, AC and BD. If carefully drawn and measured, you found that they were equal. And in general,

The diagonals of any rectangle are equal.

2. In the rectangles you have drawn, measure with your compasses the two parts into which each diagonal is divided by the other. If carefully done, you found them equal. And in general,

The diagonals of a rectangle bisect each other.

3. Carefully construct three or more squares and draw their diagonals. By use of your protractor measure the angles which the diagonals make. If carefully done, you found them all right angles. And in general,

The diagonals of a square bisect each other at right angles.

4. Draw a rectangle and cut along the diagonals into four triangles. Place those triangles that seem alike upon each other, and thus compare them.

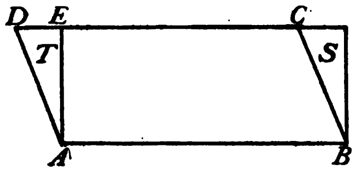
Figures that coincide throughout are **congruent**. If carefully done, you found two pairs of congruent triangles. And in general,

The diagonals of any rectangle divide the rectangle into two pairs of congruent triangles.

5. Compare the four triangles into which the diagonals of a square divide the square. State your conclusions.

4. THE AREA OF A PARALLELOGRAM

Carefully construct a parallelogram $ABCD$ of any size or shape. From point A at the extremity of the base erect a perpendicular cutting CD in E. Cut off triangle T thus formed, and place it in the position of S. What is the shape of the figure thus formed? What are its dimensions? Compare these with the dimensions of the parallelogram. If your constructions and observations were carefully made, you found that the figure formed from the parallelogram was a rectangle having the same base and altitude as the parallelogram. And in general,



A parallelogram is equal to a rectangle having the same dimensions.

Hence, $A = wl.$

1. What is the area of a parallelogram whose base is 30 ft. and whose width is 12 ft.?

2. See if you can find surfaces in the form of a parallelogram, and, if so, measure them; that is, find their areas.

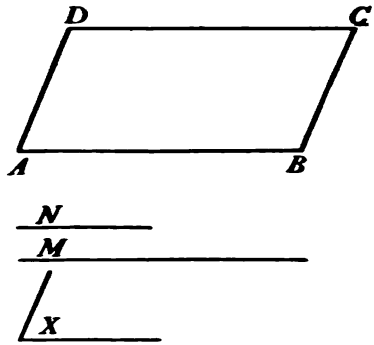
You will not find such surfaces common. A knowledge of the measure of its surface is useful on account of other surfaces being transformed into parallelograms in order to discover how to measure them.

5. CONSTRUCTIONS AND OBSERVATIONS

1. Carefully construct a parallelogram. By cutting along a diagonal, divide the parallelogram into two triangles. Compare the two triangles by placing one upon the other. If carefully done, you found them congruent. And in general,

A diagonal of a parallelogram divides it into two congruent triangles.

2. Construct a parallelogram which shall have two adjacent sides and the included angle equal respectively to two given line-segments and a given angle.



EXPLANATION. — Suppose that M and N are the given line-segments and X the given angle. The steps in the constructions are as follows:

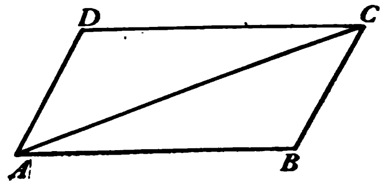
a. Construct angle BAD equal to angle X, as given in **Book I**.

b. Mark off by use of compasses $AB = M$, and $AD = N$.

c. With center at B and radius equal to N, draw an arc; then with center at D and radius equal to M, draw an arc cutting the first arc and mark the intersection C.

d. Draw BC and DC, and $ABCD$ is the required parallelogram.

3. If two forces are exerted in different directions upon the same object at A, they have the same effect as a single force called their **resultant**. If the directions and magnitudes of the two forces are represented by line-segments AB and AD, the direction and magnitude of the re-



sultant will be represented by line-segment AC, diagonal of the parallelogram $ABCD$. Construct a parallelogram to some scale and by measurement find the resultant of two forces, one of 100 lb. and the other 200 lb., acting at an angle of 60°.

4. Two forces acting at an angle of 45°, one of 40 lb. and the other of 60 lb., are equivalent to a single force of how many pounds?

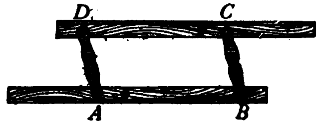
5. Draw any parallelogram and its diagonals. Compare the segments into which each diagonal divides the other. State your conclusion. Compare your conclusion with that made from a similar observation with rectangles.

6. Compare the four triangles into which the diagonals of a parallelogram divide it. State your conclusion. Compare your conclusion with that made from a similar observation with rectangles.

7. Take two pairs of strips of cardboard or light wood and join them with tacks so as to form pivots by which the form of the frame may be changed. Have the opposite sides exactly equal, as $AB = DC$ and $AD = BC$. Move about so as to form different shapes. What is the name of the figure, whatever angle the sides make with each other? This illustrates the fact that,

If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

8. The figure shows the picture of an instrument called a **parallel ruler,** used for drawing parallel lines. Study it and show why, if AB is held in a rigid position, all lines ruled along DC as it is raised or lowered will be parallel.



6. THE AREA OF A TRIANGLE

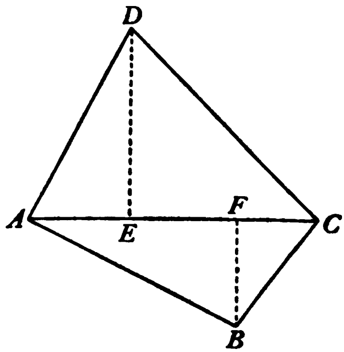
1. It was seen under the study of parallelograms that a diagonal of a parallelogram divides the parallelogram into two congruent triangles. From this fact, show how to find the area of a triangle.

2. Interpret the formula $A = \frac{1}{2}bh$ as a rule for finding the area of a triangle.

3. What is the area of a triangle whose altitude is 12 in. and whose base is 8 in.?

4. In measuring some triangular area, as a triangular plot of ground, what two measurements are necessary?

5. Draw upon the blackboard some triangle whose sides are several inches, say from 15 in. to 30 in. Now, by three different pairs of measures, find the area. That is, take each side in order as base. This will serve as a check upon the accuracy of your measurements and computation, for all results should be the same.



6. The irregular figure in the margin can be measured by dividing it up into triangles. If $AC = 30$ in., $DE = 20$ in., and $BF = 12$ in., find the area of $ABCD$.

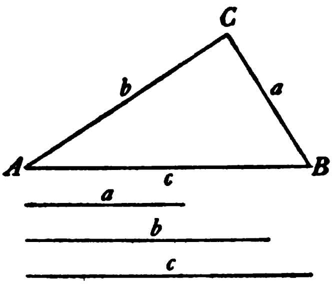
7. Make irregular figures upon the blackboard and, by making proper measurements, find the areas.

8. Draw the figure of problem 6 to a scale, making the figure several times as large as this one. Now draw diagonal BD and drop perpendiculars upon it and find the area.

7. CONSTRUCTIONS AND OBSERVATIONS

1. A triangle may be constructed with sides equal to three given line-segments a, b, and c.

The following is the order in which the construction is made:



a. Draw a straight line and lay off $AB = c$.

b. With center at A and with radius equal to b, draw an arc.

c. Then with center at B and with a radius equal to a draw an arc cutting the first arc, calling the point of intersection C.

d. Draw AC and BC, and ABC is the triangle required.

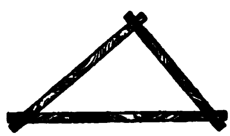
2. Construct a triangle whose sides are 2 in., 3 in., and $3\frac{1}{2}$ in., respectively.

3. Construct an isosceles triangle whose equal sides are each 4 in. and whose base is 3 in.

4. Construct an isosceles triangle whose equal sides are each equal to some line-segment x, which you have chosen, and whose base is some other line-segment y.

5. Construct an equilateral triangle each of whose sides is equal to some chosen line-segment.

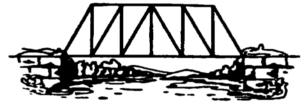
6. Nail three strips of wood together so as to form a triangle, using but one nail at each joint. Is this frame rigid, or can it be changed into various shapes by exerting pressure upon it?



This illustrates the fact that,

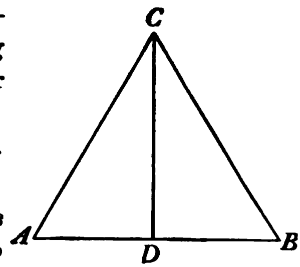
The form of a triangle is fully determined by its sides. That is, all triangles whose corresponding sides are equal are congruent.

7. Why is a roof sufficiently braced when a board is nailed across each pair of rafters?



8. Why is a long span of a bridge in which the truss is made with queen posts and diagonal rods, as shown in the drawing, sufficiently supported?

9. Draw an isosceles triangle and its altitude. Cut out the two triangles and compare them by placing one upon the other. What is your conclusion?



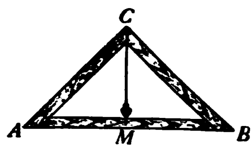
Several observations may be made. Thus:

I. *The altitude of an isosceles triangle divides the triangle into two congruent right triangles.*

II. *The altitude bisects the base and also the vertical angle.*

III. *The base angles of an isosceles triangle are equal.*

10. The drawing is that of a **plumb level**, used for leveling before the modern spirit level was invented. From a pivot at C a plumb line is hung. M is the middle point of base AB. Show how to use the plumb level in determining whether a construction is level or not.



11. Cut from cardboard any triangle and cut off the corners and place them so that the three angles form a single angle as on page 38. What is the size of the angle formed by all three angles of the triangle?

By trying this experiment with any triangle, you will find that,

The sum of the three angles of any triangle is equal to 180 degrees.

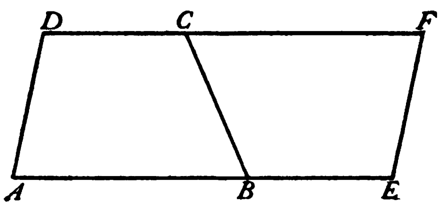
12. How many degrees in each angle of an equilateral triangle? (The angles are all equal.)

13. If the two base angles of an isosceles triangle are each 65°, what is the size of the vertical angle?

14. In a right triangle, if one acute angle is double the other, what is the size of each?

8. THE AREA OF A TRAPEZOID

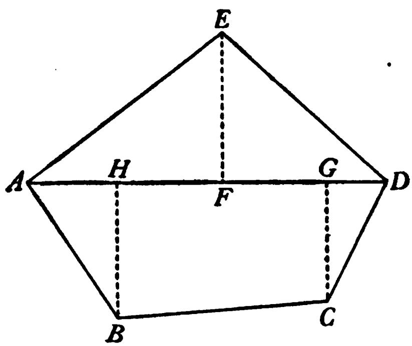
1. Take two congruent trapezoids and place them as in the figure. What kind of figure do they form? From this, state a rule for finding the area of a trapezoid.



2. State in words the relations expressed by $A = \frac{h(b + b')}{2}$, when A represents the area of a trapezoid, h its altitude, and b and b' its two bases.

3. Find the area of a trapezoid whose bases are 4 in. and 7 in. respectively, and whose altitude is 5 in.

4. This diagram is that of an irregular field. $AH = 20$ rd., $HG = 45$ rd., $GD = 15$ rd., $EF =$ 24 rd., $GC = 20$ rd., $BH = 24$ rd. Find the number of acres in it.



5. Find the area when $AH = 30$ rd., $HG = 70$ rd., $GD = 20$ rd., $EF = 50$ rd., $GC = 40$ rd., and $BH = 45$ rd.

9. THE RELATION OF THE CIRCUMFERENCE OF A CIRCLE TO ITS DIAMETER

1. Measure the circumference and diameter of several large circular objects, as dining-room tables and large wheels, or describe circles on large pieces of cardboard and cut them out for measurement. Divide the circumference by the diameter in each case. What relation do you find?

If you could have been exact enough in all your measurements, you would have found every quotient to be 3.1416. And in general,

$$\text{Circumference} = 3.1416 \times \text{diameter};$$

or,

$$C = \pi d$$

where the Greek letter π (pī) represents 3.1416, or the relation of the circumference to the diameter, and d the diameter.

2. Find the circumference of a circle whose diameter is 12 ft.

3. Find the diameter of a circle that has a circumference of 200 feet.

4. By tying a string to a stake fixed at a point that was to be the center of a circular running track, and walking about this center so as to keep the string taut, some boys laid out a $\frac{1}{8}$-mile (660 ft.) track. Find how long a string they needed besides the amount used up in tying.

5. How far does a 32-inch automobile wheel carry the automobile forward each revolution?

6. How much farther per revolution does a 36-inch wheel carry a car than a 34-inch wheel would carry it?

NOTE. — The relation found in problem 6 can be expressed as a ratio as follows: $36\pi - 34\pi = 2\pi$; $2\pi \div 34\pi = \frac{1}{17}$. Hence, the larger wheel would carry the car $\frac{1}{17}$ farther each revolution.

7. How much farther per revolution would a 32-inch wheel carry a car than a 30-inch wheel would carry it?

8. The readings of a speedometer of an automobile are controlled by the number of revolutions made by the wheels. If a speedometer is made for a car having a 32-inch wheel, and a 33-inch wheel is used, what correction must be made in the readings in order to know the actual speed or distance traveled?

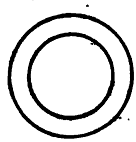
SUGGESTION. — A study of problems 6 and 7 will enable you to answer this.

9. Since $C = \pi d$, every increase in the length of the diameter gives an increase 3.1416 times as great in the circumference. When increasing the diameter of any circle, large or small, 10 inches, how much is the circumference increased?

10. On a running track having parallel sides and semicircular ends, two boys run, the outer boy being 3 ft. farther from the inner curb than the other. How much farther does he run each lap than the other boy does?

11. If one circle has a diameter 15 ft. longer than another, its circumference is how many feet longer?

12. If one circle has a circumference 15 ft. longer than another, its diameter is how much longer? Its radius is how much longer?



13. In these two *concentric circles*, if the outer one has a circumference 10 inches greater than the other, how far apart are the two circumferences? Does the size of the circles affect the answer?

14. The following question is often given to catch one: "If the earth were a smooth and perfect sphere 8000 mi. in diameter and banded at the equator with a tight fitting iron band into which a piece 12 in. long could be inserted, by how much would the insertion make the band stand out from the surface, if the space were distributed evenly around the earth?" The person asked usually says, "It would not loosen the band perceptibly." Study problems 12 and 13, then see if you cannot answer this question correctly.

10. THE AREA OF A CIRCLE

1. Describe a circle upon cardboard, taking a radius of from 4 in. to 6 in., in order to have a circle large enough to use easily.

Erect two perpendicular diameters.

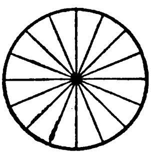
Bisect one of the four right angles formed.

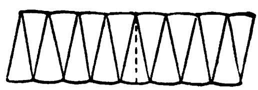
Bisect one of these two angles thus formed.

Using the arc intercepted by the angles thus formed, with your compasses divide the circumference into sixteen parts.

Now cut the circle into sixteen equal sectors as in figure A and rearrange as in figure B.

Of the figures studied, what does B most resemble?


A


B

We infer from the experiment you have made that,

The area of a circle is the same as that of a parallelogram whose base is half the circumference, and whose altitude is the radius.

Expressed as a formula,

$$A = \frac{1}{2} cr.$$

It is not necessary to measure both c and r, for if r is known, c may be found. That is, $c = 2\pi r$, hence, $\frac{1}{2}c = \pi r$. Substituting, the formula becomes

$$A = \pi r^2.$$

2. Find A when r is 12 ft.

3. Find the area of a circular flower bed 12 feet in diameter.

4. A 20-foot basin for a fountain is surrounded by a 5-foot cement walk. Find the area of the walk.

5. Compare the area of a 10-foot circle with that of a 15-foot circle.

6. The water delivered by two pipes from the same main (since both have the same pressure) varies with the areas of the cross sections of the pipes. An inch pipe will deliver how many times as much water in a given time as a $\frac{1}{2}$-inch pipe? As a $\frac{3}{4}$-inch pipe?

11. MEASURING LUMBER

In measuring lumber the unit of measure is a **board foot.** This is the equivalent of a board 1 foot square and 1 inch thick, except in measuring lumber less than 1 inch thick, in which case the thickness is not considered, but each square foot makes a board foot.

Lumber is usually sold by the thousand board feet. Thus, a quotation of "\$42 per M" means \$42 per 1000 board feet.

NOTE. — By lumbermen the term "foot" is used instead of "board foot" when no confusion in the meaning could arise from such use.

1. How many board feet in a piece of lumber 12 in. wide 14 ft. long, and 1 in. thick? That is, to how many boards 1 foot square and 1 inch thick is it equivalent?

2. How much lumber in a board 6 in. wide, 12 ft. long, and 1 in. thick?

3. How much lumber is there in a piece 6 in. wide, 18 ft. long, and 2 in. thick?

SUGGESTION. — There is just twice as much lumber in the piece as there would be if it were but 1 in. thick.

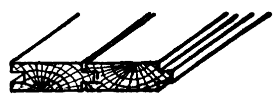
4. A beam 8 in. wide, 3 in. thick, and 18 ft. long contains how much lumber?

5. How much lumber in a piece 16 ft. long and 4 in. square?

6. A driveway to a barn is 12 ft. wide, 24 ft. long, and made of lumber 2 in. thick. Find the cost at $ 35 per M.

7. A board walk 4 ft. wide and 65 ft. long is made of lumber 2 in. thick, nailed crosswise to three pieces, each 3 in. thick and 6 in. wide, running lengthwise. Find the entire cost of the lumber at $ 38 per M.

8. Hardwood flooring is called 3-inch flooring when made from lumber three inches wide when it came from the saw mill. In making it, there is a waste of $\frac{3}{4}$ of an inch in planing and in cutting the "tongue and groove." Then how wide a strip of floor is covered by each 3-inch board?



9. Compare the $\frac{3}{4}$ inch lost in making with the $2\frac{1}{4}$ inch strip actually covered by each board.

10. The result of problem 9 may be interpreted as meaning that $\frac{1}{3}$ more lumber is needed than there is floor area to be covered. How much lumber will be needed to floor 300 sq. ft. with 3-inch flooring? To floor 450 sq. ft.? To floor 1500 sq. ft.?

11. At $ 75 per M, find the cost of the 3-inch flooring needed for a room 15 ft. by 24 ft.

12. Measure from crack to crack in the floor of your schoolroom and find what width of flooring was used.

SUGGESTION. — If the distance is $2\frac{1}{4}$ inches, 3-inch flooring was used; if $3\frac{1}{4}$ inches, 4-inch flooring was used. That is, add $\frac{3}{4}$ inch to the width from crack to crack.

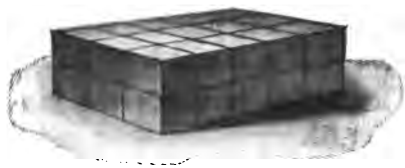
13. How much must be added to the area of the floor in your room to give the amount of lumber needed to floor it?

12. THE VOLUME OF PRISMS

You have learned that any solid whose bases are in parallel planes, and whose sides are rectangles, is a **right prism**, and that the prism is named from the shape of its base.

In a **rectangular prism** the bases, then, must be rectangles. In a rectangular prism the three edges that meet at any corner are called its **dimensions**.

1. If each division represents a foot, what are the dimensions of the rectangular prism represented here?



2. The figure represents the prism as having been cut into unit cubes. How many are here represented?

3. If we think of this prism as cut into two layers, each 3 units wide, 5 units long, and 1 unit thick, how many unit cubes in each layer? In both layers?

4. How many cubic inches in a rectangular prism 3 in. wide, 4 in. long, and 5 in. high?

5. Call a, b, and c the dimensions of a rectangular prism whose volume is V, and express the relation of V to a, b, and c by a formula.

6. State in words the truth expressed by the formula $V = abc$.

7. Find how many cubic feet of space in a coal bin 8 ft. wide, 10 ft. long, and 6 ft. deep. At 35 cu. ft. per ton, how many tons of coal will it contain?

8. What is the area of each of the six faces of a 1-inch cube? Of a 1-foot cube?

9. How many square units in the base of the prism represented in problem 1? How does this number compare with the number of cubes in each of the two layers?

10. If there are 20 sq. ft. in the base of a rectangular prism, how many cubic feet in each layer 1 ft. thick?

11. How many cubic feet in a rectangular prism whose altitude is 8 ft. and whose base contains 15 sq. ft.?

12. Let $V =$ volume, $B =$ area of base, and $h =$ height of a rectangular prism. Write the formula showing the relation of V to B and h.

13. Give in words the fact expressed by the formula $V = Bh$.

14. A rectangular watering trough 8 ft. long, $2\frac{1}{2}$ ft. wide, and 20 in. deep will hold how many gallons? (231 cu. in. = 1 gal.; 1 cu. ft. = 7.48 gal., approximately.)

15. A farmer has a bin 10 ft. wide and 12 ft. long filled with wheat to an average depth of 6 ft. How many bushels has he, allowing .8 bu. per cubic foot?

It has been found by mathematics that the number of cubic units in any prism is the product of the number of square units in the base and the number of linear units in the height. That is, $V = Bh$ is not only true of a rectangular prism, but it is true of all prisms.

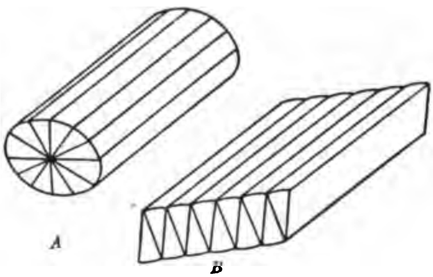
16. A concrete retaining wall 3 ft. wide at the bottom and 18 in. wide at the top is 5 ft. high and 60 ft. long. How many cubic feet of concrete are in it?

17. When water is flowing at the rate of 80 ft. per minute through a drainage ditch 20 in. wide at the bottom and 30 in. wide at the top, and at a depth of 12 in., how many cubic feet are being discharged per day?

13. THE VOLUME OF CYLINDERS

A cylinder may be divided as shown in the figure and formed into a solid closely resembling a prism, from which we infer a fact proved later in mathematics that, just as in a prism,

The number of cubic units in the volume of a cylinder is the product of the number of square units in the base and the number of linear units in the altitude.



That is, $V = Bh.$

1. A cylindrical pail 12 in. in diameter and 14 in. deep will hold how many gallons?

2. A hot-water tank 4 ft. long and 14 in. in diameter will hold how many gallons?

3. Find the capacity of a cylindrical gasoline tank 3 ft. in diameter and 8 ft. long.

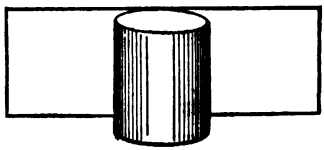
4. A hollow cylindrical iron pillar whose outer diameter is 6 in., whose inner diameter is 4 in., and whose length is 10 ft., contains how many cubic inches of iron?

5. How much water can flow in one hour, through a water pipe 2 in. in diameter, when flowing at the rate of 80 ft. per minute?

6. A cylindrical silo 16 ft. in diameter and 28 ft. high will hold how many tons of silage, allowing 50 cu. ft. per ton?

14. THE SURFACE OF A CYLINDER

The surface of a right circular cylinder consists of two circles in parallel planes, called the **bases**, and a curved surface called the **lateral surface**.



1. With a strip of paper just as wide as the height of some right circular cylinder, roll it about the cylinder, as shown in the figure, using just enough to cover the lateral area. Now, unrolling it, describe the shape and the dimensions of the paper used. State a rule for finding the lateral area of a cylinder.

2. When S represents the lateral area of a right circular cylinder whose diameter is d and whose height is h, give in words the relations expressed by the formula,

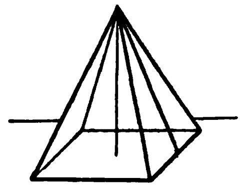
$$S = \pi dh.$$

3. Find the lateral surface of a cylinder 10 ft. long and 18 in. in diameter.

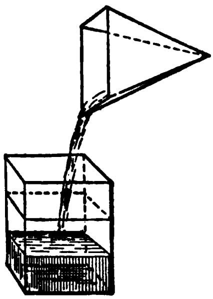
4. If a room is heated by the steam passing through 6 pipes each 14 ft. long and 2 in. in diameter (outer diameter), how many square feet of radiation are there?

5. If any room in your school is heated by cylindrical pipes, measure them and find the amount of radiation in the room.

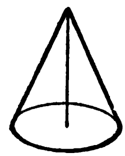
15. THE VOLUME OF PYRAMIDS AND CONES



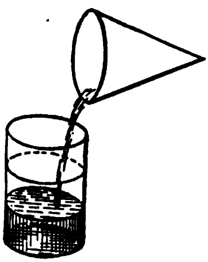
A **pyramid** is a solid bounded by any kind of polygon as **base**, and by triangles meeting at a point, called its **vertex**.



By taking a pyramid and a prism having equal altitudes and equal bases, and using the pyramid as a measure, and filling the prism, as in the figure, it will be seen that a pyramid is but one third as large as a prism of the same dimensions.



The figure in the margin is a right **circular cone**. The base is a circle. The lateral surface tapers uniformly to a point called the **vertex**. The straight line joining the vertex with the center of the base is the **altitude**. The distance from the vertex to any point in the circumference of the base is the **slant height**.



The same kind of experiment shows the same relation between a cone and a cylinder as between a pyramid and a prism.

1. Give in words the relation expressed by

$$V = \tfrac{1}{3} Bh$$

when V is the volume of a pyramid or cone, B the area of the base, and h the altitude.

2. If the base of a pyramid contains 20 sq. ft. and its altitude is 9 ft., how many cubic feet in its volume?

3. Find the volume of a cone the diameter of whose base is 12 ft. and whose height is $7\tfrac{1}{2}$ ft.

4. The base of a pyramid is 4 ft. square and its altitude is 6 ft. Find its volume.

5. A pile of sand in the form of a cone 20 ft. across the base and 9 ft. high contains how many loads (cu. yd.)?

6. A conical pile of potatoes 8 ft. across the base and 4 ft. high contains how many bushels, allowing .8 bu. per cubic foot?

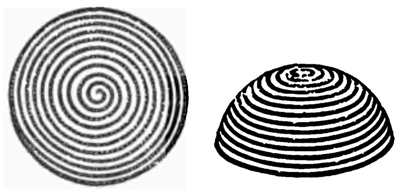
7. A conical pile of grain 12 ft. across the base and 6 ft. high contains how many bushels?

8. In one corner of a bin, a pile of wheat forms $\frac{1}{4}$ of a cone the radius of whose base is 5 ft. and whose height is 3 ft. How many bushels in the pile?

16. THE MEASUREMENT OF A SPHERE

A **sphere** is a solid bounded by a curved surface all points of which are equally distant from the **center**. A straight line from the center to the surface is the **radius**, and a line through the center terminating in the surface is the **diameter**.

A plane through the center divides a sphere into two **hemispheres**. The flat surface of a hemisphere is called a **great circle** of the sphere.



1. If the surface of a hemisphere be wound by a hard waxed cord, and that of its great circle by the same cord, it will be found that it takes just twice as much cord to wind the hemisphere as the great circle. What is your conclusion?

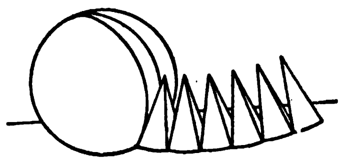
2. Where $S =$ surface of a sphere whose diameter is d, what relation is expressed by the formula

$$S = 4\pi r^2?$$

3. Find the surface of a sphere 10 in. in diameter. One 16 ft. in diameter.

4. The radius of the earth is approximately 4000 mi. How many square miles in its surface?

5. By drawing a number of planes through the center of a sphere, it may be divided into a number of solids resembling what solids that you have studied? If the bases of these solids were planes, they would be pyramids. From this we infer what is proved in later courses of mathematics that,



The volume of a sphere is the same as that of a pyramid whose base is the surface of the sphere and whose altitude is its radius.

That is, $$V = \frac{1}{3}Sr.$$

6. What is the volume of a sphere 10 ft. in diameter?

7. What is the volume of a sphere whose radius is 8 in.? Since the surface may be found from the radius, the volume depends upon the radius only. From problem 5,

$$V = \frac{1}{3} \times Sr, \text{ but } S = 4\pi r^2.$$

Substituting, $V = \frac{4}{3}\pi r^3$, the formula to remember.

8. By the formula, find the volume of a sphere whose radius is 5 in.

9. If steel weighs 490 lb. to the cubic foot, find the weight of a steel ball 6 in. in diameter.

10. A bowl in the form of a hemisphere 12 in. in diameter will hold how many gallons?

11. A cylindrical haystack 8 ft. high and 14 ft. in diameter is surmounted ("topped") with a hemisphere. Find how many tons of hay in the stack, allowing 512 cu. ft. to the ton.

SUGGESTION. — This is made up of a cylinder 8 ft. high and 14 ft. in diameter, and a hemisphere 14 ft. in diameter.

12. If a cubic foot of iron weighs 450 lb., find the weight of a hollow spherical shell 1 in. thick, with an outer diameter of 10 in.

13. Compare the volume of a sphere 4 in. in diameter with the volume of one 8 in. in diameter.

14. Compare the volume of a sphere 5 in. in diameter with that of one 15 in. in diameter.

Observe from problems 13 and 14 that the ratio of the volumes is equal to the cubes of the ratio of the diameters.

15. When one sphere has a diameter 4 times as great as that of another sphere, its volume is how many times as great?

16. An orange 4 in. in diameter is how many times as large as one 3 in. in diameter?

17. If you knew the weight of a 2 in. steel ball, how could you find, without weighing, the weight of one 10 in. in diameter?

CHAPTER IX

SQUARE ROOT AND THE PYTHAGOREAN THEOREM

IF you know the sum of two numbers and one of them, by *subtraction* you can find the other. Or if you know the product of two numbers and one of them, by *division* you can find the other. But if you know that a number is the *product of two equal numbers*, to find them requires a process called **square root.** The product is called the **square** of one of the equal numbers. Thus, the *square* of 7 is 49, and the *square root* of 49 is 7. These are written $7^2 = 49$, and $\sqrt{49} = 7$. They are read "7 squared is 49," and "the square root of 49 is 7."

1. SQUARING A TWO-FIGURED NUMBER

Subtraction and division are **inverse processes** depending upon the direct processes of addition and multiplication. In the same way, extracting the square root of a number is an **inverse process** depending upon the process of squaring a number. The general process of squaring a number may be seen by analyzing the work of squaring some number as 47.

WORK

$$
\begin{array}{rl}
47 & \\
\underline{47} & \\
329 = & 7^2 + 7 \times 40 \\
\underline{1880} = & \underline{\phantom{7^2 + {}} 7 \times 40 + 40^2} \\
2209 = & 7^2 + 2 \times 7 \times 40 + 40^2
\end{array}
$$

EXPLANATION.—It will be seen by following the work in the order in which it is done that the first step is 7×7, the next 7×40, the next 40×7, and the last 40×40.

This being the work in squaring *any* two-figured number, it is seen that,

The square of any two-figured number is the square of ones' digit plus twice the product of the ones by the value represented by the tens' digit, plus the square of the value represented by the tens' digit.

Thus, $75^2 = 5^2 + 2 \times 5 \times 70 + 70^2 = 25 + 700 + 4900 = 5625$.

By this method square :

1. 63.	**5.** 38.	**9.** 93.	**13.** 76.
2. 72.	**6.** 96.	**10.** 84.	**14.** 53.
3. 85.	**7.** 57.	**11.** 43.	**15.** 87.
4. 47.	**8.** 35.	**12.** 91.	**16.** 89.

2. FINDING THE SQUARE ROOT OF A NUMBER

Not many of the problems that you will meet in the ordinary walks of life require the process of square root, but the subject is needed in mathematics and science that you may study later and hence it is treated briefly here.

To get the process, first study the following squares to get the relation of the number of root figures corresponding to the number in the square.

Number of Figures in Roots and Powers Compared

$1^2 = 1$.	$10^2 = 100$.	$100^2 = 10{,}000$.
$9^2 = 81$.	$99^2 = 9801$.	$999^2 = 998{,}001$.

From the above powers and their roots, it appears that,

The number of periods of two figures each, beginning at ones' place, into which a whole number can be divided, equals the number of figures in the square root.

Extracting Square Root

The process is shown by the following example:

EXAMPLE. — Find the square root of 2809.

WORK

	28′09′(53
$5^2 =$	25
100	309
103	309

EXPLANATION. — It is seen that there are two root figures. The first must be 5, for $50^2 = 2500$ and $60^2 = 3600$, and 2809 lies between the two. Then, of the three addends that make 2809, 2500 or 50^2 is known. Subtracting 2500, 309 remains. This must be the sum of the other two addends, the larger of which is $2 \times 50 \times$ the ones' digit. Hence, $309 \div 100$ gives approximately the ones' digit, or 3.

Adding 3 to 100 gives 103, which multiplied by 3 gives 309, the two remaining addends being thus found by one multiplication.

Find:

1. $\sqrt{784}$.	4. $\sqrt{3136}$.	7. $\sqrt{5329}$.	10. $\sqrt{7569}$.
2. $\sqrt{3364}$.	5. $\sqrt{6889}$.	8. $\sqrt{4489}$.	11. $\sqrt{2916}$.
3. $\sqrt{8464}$.	6. $\sqrt{2704}$.	9. $\sqrt{9801}$.	12. $\sqrt{9409}$.

The process is the same for larger numbers, as shown in the following:

13. Find the square root of 2,137,444.

PROCESS

	2′13′74′44′(1462
$1^2 =$	1
2	1 13
24	96
28	17 74
286	17 16
292	58 44
2922	58 44

Find the square root of:

14. 283,024.	20. 529,984.
15. 299,209.	21. 484,416.
16. 404,496.	22. 638,401.
17. 556,516.	23. 725,904.
18. 755,161.	24. 294,849.
19. 6,017,209.	25. 1,739,761.

26. Square the following decimals: .5, .35, .245.

Observe that the square of a decimal has twice as many decimal places as the number squared.

The process of finding the square root of a decimal is shown in the following:

27. Find the square root of .734.

PROCESS

$$\begin{array}{rl}
 & .'73'40'00'\,\underline{|.856^{+}} \\
.8^2 = & .64 \\
\hline
1.6 & |\;.09\ 40 \\
1.65 & |\;\underline{.08\ 25} \\
1.70 & |\;.01\ 15\ 00 \\
1.706 & |\;\underline{.01\ 02\ 36} \\
 & \;\;.00\ 12\ 64
\end{array}$$

EXPLANATION. — Since the square root of .734 can be obtained only approximately, we plan to find it to three decimal places. Hence, zeros are added until three full periods of decimal figures are formed. Since the square of tenths is hundredths, to get the first root figure we take the first two figures at the right of the decimal point, or .73, the root of which is .8, nearly. Twice .8, or 1.6, is taken as the first divisor. Each new root figure is determined by division as in the case of integers. The inexactness of the root is expressed by + or − after the last root figure computed.

28. Study the process of extracting the square root of .501 to three decimal places and explain the steps.

$$\begin{array}{rl}
 & .'50'10'00'\,\underline{|.707^{+}} \\
.7^2 = & .49 \\
\hline
1.4 & |\;.01\ 10 \\
1.407 & |\;\underline{.00\ 98\ 49} \\
 & \;\;.00\ 11\ 51
\end{array}$$

Find the square root of:

29. .5625.	**32.** .783.	**35.** 824.9.	**38.** 1932.4.
30. .9216.	**33.** .89.	**36.** .64.	**39.** 225.9009.
31. 42.225.	**34.** 19.467.	**37.** .064.	**40.** .8.

Find the square root to two decimal places:

41. 2.	**43.** 5.	**45.** 10.	**47.** 24.
42. 3.	**44.** 7.	**46.** 18.	**48.** 39.

Methods of Using the Table

Two methods of using the table given on page 108 to find approximate roots of numbers larger than 100 are shown as follows:

Find the square root of 7235.

WORK

$$\begin{array}{r}
85.44003 \\
\underline{84.85281} \\
.58722 \\
\underline{.35} \\
293610 \\
\underline{176166} \\
.2055270 \\
\underline{84.85281} \\
85.0583370
\end{array}$$

EXPLANATION. — By the tables, $\sqrt{73} = 8.544003$. Hence, $\sqrt{7300} = 85.44003$. Also $\sqrt{72} = 8.485281$. Hence, $\sqrt{7200} = 84.85281$. The difference is .58722, which is caused by a difference of 100 between the numbers 7300 and 7200. But the given number 7235 is but 35 larger than 7200. Hence, .35 of the difference between the roots is added to the root of 7200. This is but a close approximation.

SECOND METHOD

$$\frac{7235 - 7225}{7396 - 7225} = \frac{10}{171} = .0584$$

Root = 85.0584

EXPLANATION. — The given number 7235 lies between two numbers, 7225 and 7396, whose roots are known to be 85 and 86, respectively. The difference between 7235 and 7225 is $\frac{10}{171}$ or .0584 of the difference between the two numbers, 7396 and 7225. Hence, the root is approximately that much more than 85, the root of 7225. Observe that the results by the two methods agree to three decimal places.

By the table, find the square roots to nearest hundredth:

1. 4623.	**7.** 938.	**13.** 76.2.	**19.** .3846.
2. 5781.	**8.** 722.	**14.** 84.6.	**20.** .5763.
3. 8746.	**9.** 634.	**15.** 46.7.	**21.** .936.
4. 1925.	**10.** 816.	**16.** 75.6.	**22.** .847.
5. 1478.	**11.** 738.	**17.** 35.2.	**23.** .76.
6. 3462.	**12.** 972.	**18.** 28.7.	**24.** .5.

TABLE OF SQUARES AND SQUARE ROOTS

Number	Square	Square Root	Number	Square	Square Root
1	1	1.000000	51	2601	7.141428
2	4	1.414213	52	2704	7.211102
3	9	1.732050	53	2809	7.280109
4	16	2.000000	54	2916	7.348469
5	25	2.236068	55	3025	7.416198
6	36	2.449489	56	3136	7.483314
7	49	2.645751	57	3249	7.549834
8	64	2.828427	58	3364	7.615773
9	81	3.000000	59	3481	7.681145
10	100	3.162277	60	3600	7.745966
11	121	3.316624	61	3721	7.810249
12	144	3.464101	62	3844	7.874007
13	169	3.605551	63	3969	7.937253
14	196	3.741657	64	4096	8.000000
15	225	3.872983	65	4225	8.062257
16	256	4.000000	66	4356	8.124038
17	289	4.123105	67	4489	8.185352
18	324	4.242640	68	4624	8.246211
19	361	4.358898	69	4761	8.306623
20	400	4.472136	70	4900	8.366600
21	441	4.582575	71	5041	8.426149
22	484	4.690415	72	5184	8.485281
23	529	4.795831	73	5329	8.544003
24	576	4.898979	74	5476	8.602325
25	625	5.000000	75	5625	8.660254
26	676	5.099019	76	5776	8.717797
27	729	5.196152	77	5929	8.774964
28	784	5.291502	78	6084	8.831760
29	841	5.385164	79	6241	8.888194
30	900	5.477225	80	6400	8.944271
31	961	5.567764	81	6561	9.000000
32	1024	5.656854	82	6724	9.055385
33	1089	5.744562	83	6889	9.110438
34	1156	5.830951	84	7056	9.165151
35	1225	5.916079	85	7225	9.219544
36	1296	6.000000	86	7396	9.273618
37	1369	6.082762	87	7569	9.327379
38	1444	6.164414	88	7744	9.380831
39	1521	6.244998	89	7921	9.433981
40	1600	6.324555	90	8100	9.486833
41	1681	6.403124	91	8281	9.539392
42	1764	6.480740	92	8464	9.591663
43	1849	6.557438	93	8649	9.643650
44	1936	6.663249	94	8836	9.695359
45	2025	6.708203	95	9025	9.746794
46	2116	6.782330	96	9216	9.797959
47	2209	6.855654	97	9409	9.848857
48	2304	6.928303	98	9604	9.899494
49	2401	7.000000	99	9801	9.949874
50	2500	7.071067	100	10000	10.000000

3. SOME APPLICATIONS OF SQUARE ROOT

Some of the indirect problems of mensuration require square root. Thus, the area of a square whose side is 47 in. is 47×47, or 2209 sq. in. But the number of inches in the side of a square containing 2209 sq. in. is $\sqrt{2209}$ or 47. Likewise, the area of a circle whose radius is 15 in. is $15 \times 15 \times 3.1416$, or 706.86 sq. in.; but the number of inches in the radius of a circle having an area of 706.86 sq. in. is $\sqrt{706.86 \div 3.1416}$, or $\sqrt{225}$, which is 15.

1. Find the side of a square containing 5329 sq. in.

2. Find the dimensions of a rectangle twice as long as it is wide that contains 8978 sq. in.

Suggestion. — The rectangle will make two squares each containing 4489 sq. in. Show this by a diagram.

3. Find the diameter of a circular plot that has the same area as a rectangular one 18 ft. by 24 ft.

4. What must be the diameter of a circle having three times the area of one 10 ft. in diameter?

5. What must be the side of a square having twice the area of one whose sides are each 16 ft.?

6. A water main 15 in. in diameter is to be replaced by one having three times the carrying capacity. What must the diameter of the new pipes be?

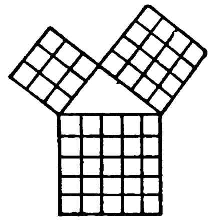
Suggestion. — To have three times the carrying capacity, the area of a cross section must be three times as large as the area of the smaller.

7. Frank has a garden 38 ft. by 96 ft. If he replaces this by a square garden of the same area, what must be its side?

8. How much less fencing is needed to inclose a square garden as large as a rectangular one 120 ft. by 300 ft.?

4. THE PYTHAGOREAN THEOREM

A **right triangle** is a triangle of which one angle is a right angle. The side opposite the right angle is called the **hypotenuse**, and the other sides are called the **legs**. By drawing a right triangle whose legs are 3 in. and 4 in., respectively, it will be seen that the hypotenuse is just 5 in., and that the area of the square on the hypotenuse equals the sum of the areas of the squares on the two legs.

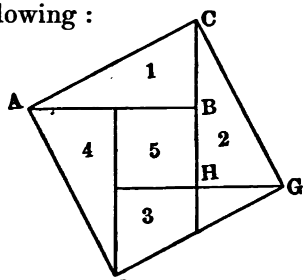


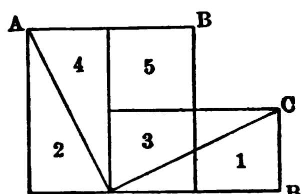
This important truth was proved by Pythagoras, about 500 B.C., to be true of any right triangle. That is,

The square on the hypotenuse of any right triangle is equal to the sum of the squares on the other two sides.

NOTE. — Carpenters make use of this fact in laying out the foundation of a building when they want the walls at right angles to each other. Starting at one corner, a line 8 ft. long is taken in one direction along which the foundation is to be laid. Starting from the same corner, another line 6 ft. long is fastened to the end of the first line and moved about until a 10-ft. rod will just reach the outer extremities of the two lines.

The truth of the Pythagorean theorem may be seen by drawing, or cutting from cardboard, figures like the following:





Let ABC be the right triangle. The square on the hypotenuse AC is equal to the four triangles, 1, 2, 3, and 4, and the small square, 5. Now put 1 and 2 in the position of the figure at the right, and the figure is equal to a square on AB and one on CB.

1. One leg of a triangle is 48 ft. and the other is 36 ft. What is the hypotenuse?

2. The hypotenuse of a right triangle is 85 ft. and one leg is 51 ft. What is the other leg?

3. One leg of a right triangle is 76 ft. and the hypotenuse is 95 ft. What is the other leg?

4. What is the diagonal (distance between the opposite corners) of a rectangle 92 ft. long and 69 ft. wide?

5. How long is the diagonal of a 30-foot square?

6. What is the length of the longest straight line that can be drawn on a sheet of paper 16 in. by 20 in.?

7. How far is a place 12 mi. east of you from one 18 mi. north of you?

8. What is the distance between the opposite corners of a field 200 rd. long and half as wide?

9. If a window is 18 ft. from the ground, how long must a ladder be to reach to the window if the foot of the ladder is placed 6 ft. out from the building?

10. In decorating a room two ribbons are stretched, connecting the opposite corners. If the room is 30 ft. wide and 40 ft. long, how many yards of ribbon does it take?

11. A baseball diamond is 90 ft. square. How long is the throw from first to third base?

12. A derrick is 48 ft. high, and is supported by three steel cables, each reaching from the top of the derrick to a stake in the ground 45 ft. from the foot of the derrick. How

much steel cable does it take, allowing 10 ft. for fastening all three cables?

13. For reaming round holes .5 of an inch in diameter, a square reamer is used. Find the dimensions of the reamer.

14. If the base of an isosceles triangle (a triangle having two equal sides) is 12 in. and its altitude is 10 in., find the length of its equal sides.

SUGGESTION. — The altitude of an isosceles triangle divides the triangle into two equal right triangles.

15. If the equal sides of an isosceles triangle are each 15 in. and the base 12 in., what is the altitude?

16. Find the area of an isosceles triangle whose base is 14 in. and whose equal sides are each 12 in.

17. Find the area of an equilateral triangle each of whose sides is 20 in.

CHAPTER X

GENERAL DISCUSSION OF PERCENTAGE

You have become familiar with the use of **per cent** to express the relations between two numbers, and have seen that it is but another name and notation for *hundredths*. Thus,

$$8\% = \tfrac{8}{100} = .08; \qquad 2\tfrac{1}{2}\% = .025;$$
$$.28 = \tfrac{28}{100} = 28\%; \qquad .035 = 3\tfrac{1}{2}\%.$$

Problems whose relations are expressed in per cent are sometimes called **percentage problems.**

1. A REVIEW OF FORMER WORK IN PERCENTAGE

In the mathematics that you have had, you had but two kinds of problems involving the use of per cent. They were:

(1) *To find a per cent of some number;* and

(2) *To find what per cent one number is of another.*

These two problems are by far the most common in all ordinary uses of mathematics. As you have seen, the first is but an application of the multiplication of decimals after the per cent has been expressed as a decimal; and the second is but an application of the division of decimals, with the exception of expressing the quotient as a per cent.

Drill Exercises

Change to decimals:

1. 45%.	6. 16%.	11. 15.4%.	16. $4\frac{1}{2}$%.
2. 63%.	7. $16\frac{1}{2}$%.	12. 3.6%.	17. $6\frac{1}{4}$%.
3. 7%.	8. 4%.	13. .8%.	18. $8\frac{3}{4}$%.
4. 9%.	9. $4\frac{1}{2}$%.	14. 125.4%.	19. $9\frac{1}{2}$%.
5. 138%.	10. 240%.	15. 245.6%.	20. 200%.

Find:

21. 24% of 650.	27. 125% of 980.	33. 2.4% of 780.
22. 38% of 98.5.	28. 240% of 650.	34. 24.5% of 540.
23. 6.4% of 760.	29. 350% of 920.	35. 2.45% of 360.
24. 5.2% of 84.5.	30. 175% of 685.	36. $17\frac{1}{4}$% of 560.
25. 21.5% of 780.	31. $16\frac{1}{2}$% of 840.	37. $37\frac{1}{2}$% of 560.
26. 9.5% of 865.	32. $4\frac{3}{4}$% of 720.	38. 200% of 765.

Change to per cent:

39. .35.	43. .735.	47. 1.35.	51. .015.
40. .48.	44. .864.	48. 2.48.	52. .008.
41. .09.	45. .025.	49. 2.9.	53. .0085.
42. .095.	46. .258.	50. 3.2.	54. .1225.

Find what per cent:

55. 48 is of 85.	58. 46.8 is of 39.6.	61. 9 is of 4.48.
56. 95 is of 148.	59. 175 is of 84.5.	62. 1.8 is of .96.
57. 7.3 is of 16.5.	60. 28.7 is of 36.	63. 7.2 is of 4.35.

2. INTERPRETING AND FINDING PER CENTS OF INCREASE OR DECREASE

In general reading we constantly meet references to increases or decreases in production, consumption, prices, wages, and in various other things in which we are interested, all given in terms of per cent. To read intelligently such articles we must be able to interpret such references and to find such relations for ourselves.

1. In a recent news item it was stated that from 1914 to 1918 the cost of food had advanced 54 % and of clothing 66 %. What does this mean? Food that cost $1 in 1914 would cost how much in 1918 at this rate of increase? Clothing that cost $20 in 1914 would cost how much in 1918?

2. *The Literary Digest*, Sept. 7, 1918, says that an unmistakable evidence of thrift among wage-earners is shown by the fact that the membership in Building and Loan Associations has increased 52 % since the beginning of the War in 1914. What does this mean? To every 100 members in 1914 there were how many in 1918?

3. The same article (problem 2) states that the increase in the amount of deposits during the 15 years preceding 1918 was 205 %. What does this mean? To every $100 deposited fifteen years before, how many dollars were deposited in 1918?

4. The same article (problem 2) says that while the membership increased 52 % during the four years preceding 1918, the amount of the deposits increased but 30 %. From this, are the average individual deposits larger or smaller than those four years ago?

5. The Bureau of Immigration reported that our immigration had lost 78% from January, 1917, to January, 1918, and that it had lost 86% since 1913. What does this mean? To every 100 immigrants landing in 1913, how many landed in 1918? To every 100 landing in 1917, how many landed in 1918?

6. Our potato crop averages but 90 bushels per acre, while that of France averages 135 bushels, and that of Great Britain averages 124 bushels. The production of potatoes in each of these countries is an increase of what per cent over the average amount that we produce per acre?

7. A magazine article in June, 1918, says that the fact that the price of imports into and exports from the United States has increased from 50% to 100% within a year shows the general advances in prices to be world-wide. What does an increase of 50% to 100% mean?

8. Olive oil was imported from Italy at an average price of $1.25 per gallon in 1914, and of $3.05 per gallon in 1918. Find the per cent of increase in price.

9. Flax was imported at $290 per ton in 1914, and at $1188 per ton in 1918. Find the per cent of increase. Linen being made from flax, what would such an increase indicate as to the cost of all linen articles?

10. We exported upland cotton at an average price of 8.5¢ per pound in 1915, and at an average price of 31.7¢ per pound in 1918. Find the per cent of increase.

11. A magazine article of August, 1918, says, "That submarine warfare has still a long way to go to stop or even check our trade with the rest of the world, is shown by the following report:"

	1915	1916	1917
Exports	$2,500,041,944	$3,867,115,373	$5,718,000,000
Imports	1,516,474,600	1,952,033,212	2,342,000,000

Find the rate of increase each year over the preceding, both in exports and in imports.

12. During the War there was a marked decrease in the importation of many articles of food usually classed among the luxuries. There were but 9,000,000 pounds of cheese imported in 1918 as against 15,000,000 pounds in 1917, and 64,000,000 pounds in 1914; the import of currants in 1918 was but 5,000,000 pounds as against 25,000,000 pounds in 1916, and 32,000,000 pounds in 1914; and the importation of dates dropped from 34,000,000 pounds in 1914 to 6,000,000 pounds in 1918. Find the per cent of decrease of each article from one date to the next.

3. A NEW PROBLEM IN PERCENTAGE

Since a per cent expresses the relation of one number to another, if one number and its relation to the other, expressed in per cent, are known, the other number may be found. Thus, if 35 % of some number is known to be 70, the number is evidently 200. For 35 % of 200 is 70. This, then, is the inverse of the first kind of problems and is called the **indirect problem of percentage.** This type of problem has fewer practical applications than the two kinds already studied. However, it has a use; hence, the way to solve it will be shown here. The general problem is,

To find all of a number when a certain per cent of it is known.

This problem is easily recognized; for the per cent named in the problem refers to a number not given in the problem,

but to the one to be found, instead of referring to the number given, as in the first type of percentage problem studied.

1. An article costing $23.10 will have to sell for what price in order to give the dealer a gross profit of 40 % of the selling price?

ANALYSIS OF THE PROBLEM. — Our experience tells us that when a dealer sells an article at a profit, he is getting back, in the selling price, both the cost and the profit. That is, the selling price equals the cost plus the profit. But since 100 % of anything is all of it, if 40 % of the selling price is profit, the remaining 60 % of it must represent the cost. Hence, we have the following relation:

60 % of the selling price = $23.10.

This means .6 × (an unknown factor) = $23.10. That is, the product of two factors and one of the factors are known. The problem is to find the other. From the meaning of division the solution is

$23.10 ÷ .6 = $38.50.

2. An article costing $52.50 must sell at what price to give a profit of 30 % of the selling price?

SOLUTION

.7)$52.50
$75

EXPLANATION. — Since 30 % of the selling price was profit, 70 % of it must be the cost, or $52.50. Then $52.50 ÷ .70 must give the selling price.

3. At what price must goods be marked in order that 20 % of the marked price may be deducted and leave a selling price of $7.60? (80 % of marked price = $7.60.)

4. At what price must goods be marked if $5.60 is to be received for them after deducting 30 % of the marked price?

5. A merchant having mislaid the cost price of some goods marked at $9.75 remembers that they were marked to sell at 30 % above the cost. Analyze the problem and show how he can find what the goods cost him.

6. In making white flour, 72 % of the wheat is used. How many bushels (60 lb.) of wheat will it take to make 12 bbl. (196 lb. per bbl.) of wheat flour?

7. In making whole wheat flour, 85 % of the wheat is used. How many bushels of wheat will it take to make 12 bbl. of whole wheat flour?

8. In making graham flour, 95 % of the wheat is used. Find how many bushels of wheat are needed to make 12 bbl. of graham flour.

9. Experience shows that cattle lose 44 % of their live weight in dressing. What must be the live weight of cattle that dress 616 lb.? That dress 845 lb.? That dress 580 lb.?

10. When small undressed fish that will lose 45 % in dressing are selling at 18 ¢ per pound, a slice of a larger fish having no waste is selling at 24 ¢ per pound. Which will cost less and how much less when 5 lb. of dressed fish are needed?

11. Hogs lose 20 % in dressing. How large a hog will it take to dress 192 lb.?

12. About 20 % of the dressed weight of a hog goes into lard. A farmer got a 48-pound pail of lard from one hog. How large must it have been?

13. In canning berries a woman found she got an average of but 40 % as much canned fruit as she used of raw fruit. How many berries must she buy for 48 quarts of canned fruit?

14. If a dealer sells you an article for $24 which he tells you is 20 % less than the usual price, find the usual price.

15. During a special sale, a firm sold all of its athletic goods for 10 % less than the regular price. How much would you have to pay at regular price for what you could buy for $14.85 during the special sale?

Drill Exercises

1. 115.2 is 20 % more than what number?

2. 76.8 is 20 % less than what number?

3. $10.92 is 40 % more than what sum?

4. $4.68 is 40 % less than what sum?

5. A gain of $4.50 is 20 % of what an article cost. Find the cost.

6. A gain of $7.50 is 15 % of what goods sold for. Find the selling price.

7. When selling goods for $10.08, 20 % of the cost is gained. Find the cost.

8. When goods costing $8.75 are to sell at a profit of 30 % of the selling price, find the selling price.

9. When chickens lose 25 % in dressing, what size, live weight, will dress $1\frac{1}{2}$ pounds?

10. A dressed hog weighs about 80 % of its live weight. What live weight will dress 280 pounds?

4. APPLICATIONS OF THE THREE PROBLEMS OF PERCENTAGE

Most of the relations expressed about the quantitative side of life are expressed in terms of per cent. In our daily reading we see increases or decreases of all kinds referred to in per cent. To enable you to interpret such references is the purpose of this list of problems.

1. It is estimated that a man having a family of five and receiving a salary of $3500 per year should portion his several expenses as follows: food, 25 %; rent, 20 %; clothing, 22 %; operating expenses, 15 %; and use the balance for savings, charity, and recreation. Find how much this would allow for each item.

2. It is estimated that a family of four, with an income of $1500 per year, should divide it as follows: food, 35 %; rent, 20 %; operating expenses, 15 %; clothing, 18 %; and use the balance for savings, charity, and recreation. Find how much this would allow for each item.

3. A study made in 1907 showed that untrained girls were earning a maximum of $8.75 per week, while the trained girls of the same age were earning $20.25. Find what per cent more the trained girls were earning.

4. In 1909 it was found that at the age of thirty the average salary of a group of men taken at random, all of whom had received a grammar school education, but no further training, was $1253, while the average yearly wage of a number of illiterate workers at the same age was but $500. The first group received how many per cent more?

5. Statistics show that at the end of 1917 the average increase in price of food since the end of 1915 had been 63 %. If so, $100 at the end of 1917 would buy as much as what sum in 1915? It would take how much in 1915 to buy as much as $100 would buy in 1917?

6. The increase in price of six important commodities is shown in the following table. Compute the per cent of increase in the price of each commodity.

Crop	Unit	1916	1917
Wheat	bushel	$1.071	$2.289
Corn	bushel	.794	1.966
Barley	bushel	.593	1.145
Rye	bushel	.834	1.781
Potatoes	bushel	.954	1.708
Cotton	pound	.126	.243

7. In 1916 we produced but 640,000,000 bushels of wheat. This was a decrease of what per cent of a five-year average of 728,000,000 bushels?

8. In 1915 we produced 1,025,000,000 bushels of wheat. This was an increase of what per cent over a five-year average of 728,000,000 bushels?

9. What is meant by saying that the production of a certain article is 225 % of its former production?

10. What is meant by saying that the production of an article has increased 225 % over its former production?

11. If the factory output of a certain article is now 225 % of its former output of $725,000 yearly, what is it now?

12. If the factory output of a certain article is now 225 % more than its former output of $725,000 yearly, what is it now?

13. If in 1918 we decreased our average consumption of wheat, which was 580,000,000 bushels yearly, by 27 %, what was the consumption of that year?

14. Before the World War, France used 380,000,000 bushels of wheat annually, 85 % of which she produced. If her production was cut 40 % during the War, how much did she have to import annually to give the same consumption?

15. A news item says, "Montana, Idaho, Wyoming, and Oregon produced 86,255,000 pounds of wool in 1916, which is about 30 % of the total production of the entire United States." From these data find the total production of that year.

16. With an increase in the acreage under cultivation and the consequent restricting of pasturing acreage, the number of sheep raised in Texas decreased from 4,260,000 in 1890 to 1,600,000 in 1915. Find the per cent of decrease.

17. Vermont at one time was a very large sheep-producing state, but the number decreased from 1,682,000 in 1840 to 47,000 in 1915. The number in 1915 was what per cent of the number in 1840?

18. Rye being hardier than wheat and succeeding in poorer soils, the Department of Agriculture in 1917 recommended an acreage of 5,131,000. "If planted, this will be an increase of 22 % over our ten-year average," says a news item. From these data find our ten-year average.

19. Our acreage of beans, an especially important food in war time, was 84 % more in 1917 than in 1916. If you knew the acreage in 1916, how could you find the acreage of 1917? If you knew the acreage of 1917, how could you find the acreage of 1916?

20. If a merchant pays $24.50 for an article and marks it so as to give a discount of $5.50 from the marked price and still make 20 % of the marked price, find the price at which he marked it. The discount was what per cent of the marked price?

21. If fish lose 40 % in dressing, what is the cost per pound of the dressed fish when undressed fish are 18 ¢ per pound?

22. Which is cheaper and how much: live chickens at 25 ¢ per pound, or dressed ones at 35 ¢, if the loss in dressing is 30 %?

23. What per cent of his sales is the ice man making if he sells ice at 60 ¢ per 100 pounds, for which he pays, including the cost of delivery, $7.50 per ton, the loss through melting being 15 % of each ton?

CHAPTER XI

BUSINESS TERMS, FORMS, AND PROBLEMS

IN **Book I** you studied certain common **business terms** and **forms** that you meet in the ordinary walks of life. These will be reviewed and extended so that you will be able to interpret references to them, which you will meet more and more in general reading and in conversation.

1. BILLS RENDERED BY THE RETAIL MERCHANT

Bills are statements rendered to a purchaser showing the date and price of purchases made, credits if any, and the final amount due. To keep one's credit with a store, all bills should be settled promptly. That is, within a few days of the time they were rendered.

1. If you bought goods, paying \$3 at the time of purchase, and returned goods costing \$1.25, what is the total credit allowed on the bill?

2. If your mother buys $6\frac{1}{2}$ yd. of cloth at 95¢ per yard, 5 yd. of lining at 65¢ per yard, a house dress at \$3.85, and a waist at \$2.98, find the total amount of the bill.

3. If, in problem 2, the dress is returned, what credit item will the bill show when the bill is rendered, the goods having been charged to her account?

4. Check the following bill, that is, see if there is any error in the computation:

1918	SEPT.			ITEMS	TOTAL CHARGES	TOTAL CREDITS	BALANCE
4224	3	1 dress			$22.98		
4906	6	$4\frac{1}{2}$ yd. satin	$3.50	$15.75			
1128		$4\frac{1}{2}$ yd. satin	2.75	12.38			
926		3 yd. percale	.59	1.77	29.90		
3107	8	1 skirt			3.98		
2986	13	2 yd. lining	.59	1.18			
1732		1 waist		2.98			
4102		$\frac{1}{2}$ yd. silk	2.50	1.25			
4396		1 pr. gloves		1.75	7.16		
	14	1 skirt retd.				$3.98	
4510	17	$12\frac{1}{2}$ yd. braid	.20	2.50			
3946		1 bag		3.98			
4098		$\frac{1}{2}$ doz. buttons	1.25	.63	7.11		
					$71.13		$67.15

5. Rule paper and make out a bill for the following, heading it with the name of some store in your city and naming Mrs. Richard Roe as buyer: 3 pr. of hose at 69¢; 2 skirts at $4.25; 6 yd. satin at $3.98; 4 yd. lining at 59¢; $1\frac{3}{4}$ yd. belting at 32¢; $6\frac{1}{2}$ yd. silk at $3.25; 1 suit case at $9.75; $4\frac{3}{4}$ yd. satin at $3.25; credit 1 skirt returned, $4.25; and $10 cash payment at time of purchase.

6. Why should one keep a receipted bill, that is, a bill that has been paid? Sometimes, when payment has been made by a check on some bank, the top part of the bill showing the name of the purchaser and the amount of the bill is torn off and that part only is returned with the payment, the part shown in problem 4 being kept by the purchaser. In such cases, no receipt of the paid bill is returned. See if you can find out from some older person why no receipt is necessary.

7. Bring to class paid bills. Tell of whom the goods were purchased, by whom, and when payment was made. Check them to see if any error was made in computation.

2. KEEPING ACCOUNTS

Careful persons in all walks of life keep some sort of account of their business dealings. An **account** is a record of value received and of value delivered. These accounts are kept in various forms, depending upon the needs of those keeping them. A few forms are shown here.

Personal Cash Accounts

One should early form the habit of keeping a careful record of all money received and when and for what it was expended. The following simple form is a very common type.

PERSONAL CASH BOOK

1919			RECEIVED		PAID	
Jan.	6	Balance on hand	4	60		
	7	Received allowance	2	00		
	7	Paid for book			1	25
	8	Paid for lunch				30
	8	Received for errand		50		
	9	Received for shoveling snow		75		
	10	Paid for R.R. ticket			1	20
	12	Deposited in savings bank			4	00
	12	Balance			1	10
			7	85	7	85
Jan.	13	Balance on hand	1	10		

The *balance item* is entered in the smaller column, so that each side will "total" the same. It is the excess of the total amount received over the total amount paid out.

1. Rule paper like the form shown here and make out a "cash account" showing the balance at the end of each week. Receipts: March 5, 1919 (Monday), balance on hand, $3.20; March 6, allowance, $1; March 8, errands, 60¢; March 9, payment for bicycle, $15; March 10, errands, 90¢; March 13, allowance, $1; March 15, balance on bicycle, $10; errands, 40¢; March 17, for delivering papers for the week, $2.75; March 20, allowance, $1; March 21, for 8 hens, $14.30; March 24, errands, 70¢; March 24, delivering papers for the week, $2.75. Paid: March 8, carfare, 20¢; March 9, lunch, 30¢, carfare, 10¢; March 14, lunch, 35¢, magazine, 20¢; March 16, carfare, 20¢; March 17, deposited in savings bank, $9; March 22, bought 4 hens, $7.10; March 23, feed for hens, $1.50; March 24, deposited in savings bank, $5. Show the balance at each week-end and the balance on hand Monday, March 26.

2. Arrange the following as a cash account balanced at the end of each week. It begins Apr. 1, 1919 (Tuesday), hence, balance it Apr. 5, 12, 19, and 26, leaving the last items, beginning Apr. 28, unbalanced: 1. Cash on hand, $4.50, spent 15¢ carfare, 30¢ for lunch; 2. Allowance, $1, from errands, 40¢; 3. Bought magazine, 20¢, had bicycle repaired, $1.20; 5. Deposited $3 in savings bank, received $3.25 for delivering papers; 7. Allowance, $1, carfare, 20¢; 8. From errands, 60¢, for lunch, 30¢, carfare, 10¢; 9. Movie ticket, 20¢, carfare, 10¢; 11. Bought catcher's glove, 85¢, carfare, 10¢; 12. Received the week's wages for delivering papers, $3.25, deposited $3 in savings bank; 14. Allowance, $1, sold old catcher's glove, 35¢; 16. Errands, 50¢, carfare, 15¢; 17. Bought magazine, 20¢, fruit, 10¢; 19. Week's wages for delivering papers, $3.25, deposited, $3; 21. Allowance, $1, for carrying packages, 65¢; 22. For

raking lawn, 50 ¢; 23. Carfare, 10 ¢; 25. From errands, 50 ¢; 26. Week's wages for delivering papers, $ 3.25, deposited, $ 4; 28. Allowance, $ 1; 29. Earned 60 ¢ delivering packages, spent 30 ¢ for lunch; 30. Movie ticket, 20 ¢, carfare, 10 ¢, fruit, 15 ¢.

3. Keep a cash account of your own receipts and expenditures for a month, and bring it to class one month from the time you study this.

Household Accounts

A **monthly household account** should show a record of all incomes for the month, as salary and other items, and of all expenses, as items for rent, food, operating expenses (fuel, lights, wages, etc.), clothing, and higher life (amusements, travel, church, charity, education, etc.). The usual form is that of the personal cash account which you have studied. The items in the monthly account are taken from "totals" shown in the daily and weekly accounts.

1. Rule paper and balance the following, supplying dates: Receipts: balance of cash on hand, $ 48.50; salary, $ 225; miscellaneous, $ 30.40. Paid out: rent, $ 50; food, $ 48.75; clothing, $ 52.80; operating expenses, $ 14.30; higher life, $ 28.30; savings bank deposit, $ 30.

2. Balance the following: Cash on hand, $ 28.30; monthly allowance for expenses, $ 150. Paid out: food, $ 45; clothing, $ 38.50; operating expenses, $ 16.80; rent, $ 35; higher life, $ 6.80; health, $ 3.75.

3. Balance the following: Cash on hand, $ 32.75; monthly allowance, $ 125. Paid out: rent, $ 30; food, $ 42.50; clothing, $ 34.20; operating expenses, $ 10.30; higher life, $ 4.25.

4. In the following weekly account find : (*a*) the total of each item for the week ; (*b*) the total expense for each day ; and (*c*) the total expense for the week.

WEEKLY HOUSEHOLD EXPENSE ACCOUNT

	MON.	TUES.	WED.	THURS.	FRI.	SAT.	SUN.	TOTAL
Food								
Groceries	$2.10		$ .90			$1.20		
Meat	.70	$ .30	.50	$ .40	$ .30	1.30		
Milk	.16	.16	.16	.32	.16	.32		
Clothing			4.80			6.30		
Operating expenses								
Fuel	4.50							
Light						1.20		
Telephone						.40		
Laundry	1.80							
Higher life								
Amusements			.50			1.00		
Papers and magazines	.02	.02	.02	.02	.07	.04	$ .07	
Church							1.00	
Charity	.50							
Health		2.50						
Total								

Ledger Accounts

In business, more elaborate accounts are kept. There are two terms that the business man employs that were not used in the simple accounts that you have studied. These two terms are **debit** and **credit**. A record of *debits* is a record of debts or of value received ; a record of *credits* is a record of value delivered. Among the most common accounts in business are *personal accounts ; cash accounts ; merchandise accounts ;* and *expense accounts.*

A merchant's personal account shows the amount owed to or owed by the person whose name appears at the head of the account.

The following shows the form of ledger account between L. Harris & Sons (merchants) and J. S. Lee, a customer.

Dr. J. S. LEE, 56 ELM ST. *Cr.*

1919					1919				
Jan.	3	Mdse.	28	40	Jan.	6	Cash	25	00
	8	"	16	70		15	"	15	00
	28	"	19	80	Feb.	1	Balance	24	90
			64	90				64	90
Feb.	1	Balance	24	90		10	Cash	30	00
	6	Mdse.	17	80		20	"	20	00
	15	"	26	30	Mar.	1	Balance	19	00
			69	00				69	00

1. Check the personal account shown above, that is, see if the computation is correct.

2. Who bought the merchandise shown by the items of the account? Of whom were they bought? Who kept the account?

3. What does the balance on Feb. 1 show? On March 1? Had there been a balance on the "Dr." side, what would it have shown? In such an account is there likely to be a balance on that side? Why?

4. Had the merchant loaned Mr. Lee $15, upon which side would the item have been written? Why?

5. Had Mr. Lee sold the merchant some produce, or rendered him some service, upon which side of the account would it have been placed? Why?

6. Pretending that you are a merchant and that some student is your customer, make up and balance an account, and explain the meaning of each item.

A merchant's **Cash account** is a record of debits and credits of cash, the *balance* showing the cash on hand. It is as if the merchant was keeping an account with his own cash box. Hence, "Cash" is debtor of all that is put into it, and credited with all that is taken out.

Dr. CASH *Cr.*

1919					1919				
May	1	Balance	986	30	May	7	Mdse.	1250	00
	5	Mdse.	680	50		10	"	750	00
	7	"	950	00		10	Office furniture	150	00
	15	J. Morris	250	00		15	Wages	200	00
						15	Balance	516	80
			2866	80				2866	80
May	16	Balance	516	80					

7. Do the "mdse." items of May 5 and 7 show a record of merchandise bought or sold? Why?

8. What does the "J. Morris" item of May 15 show?

9. What do the merchandise items on the credit side show?

10. What is the meaning of the item "office furniture" of May 10?

11. Check the account and see if any mistakes have been made in the computation.

The **Merchandise account** is a record of the cost of goods bought (debits) and of the receipts from goods sold (credits). The **inventory** item on the credit side shows the value of the goods on hand. The *balance* shows whether there was a loss or gain.

Dr. MERCHANDISE *Cr.*

1919					1919				
June	1	On hand	840	00	June	5	Cash	576	80
	10	Brown & Co.	960	40		7	Note	240	00
	12	R. L. Smith	796	30		18	S. C. Hart	160	00
	18	Cash	536	80		24	Cash	1080	00
	30	Balance	448	60		30	Inventory	1525	30
			3582	10				3582	10
July	1	On hand	1525	30					

12. Check the merchandise account shown here.

13. What items show the amount of goods bought? Of goods sold? Of goods unsold?

14. What does the balance of $448.60 on the debit side show? What would it have shown had it been on the credit side?

15. What does the "note" item of June 7 show? The "S. C. Hart" item of June 18th?

16. What do the "cash" items on the credit side show? The "cash" item on the debit side?

The **Expense account** shows the cost of doing business. To the account, rent, fuel, lights, postage, salaries, etc., are debited, and unused coal, postage, etc., are credited in finding the balance, which is the net cost of doing business.

Farm Accounts

Successful farmers often keep an account with each crop or kind of stock raised, as *account with wheat; account with corn; account with hogs*, etc. More often they are like the following form:

ACCOUNT WITH WHEAT, 40 ACRES

			Cost		Returns	
1918	Oct.	Plowing and seeding	170	00		
		Seed	225	00		
	June	Cutting	70	00		
	Aug.	Threshing	40	00		
		Interest on land investment	240	00		
		820 bu. @ $2.20			1804	00
		50 tons straw @ $4.50			225	00

17. Find the net profit per acre from the wheat.

Rule forms for a merchant's personal account with his customers and balance the following:

18. Roberts & Sons in account with R. L. Jones. Sales: Sept. 3, furniture, $386; Sept. 5, rugs, $175; table, $48; Sept. 15, refrigerator, $38, range, $48. Credits: Sept. 3, cash, $250; Sept. 10, cash, $150.

19. Cuthbertson Bros. in account with W. A. Miller. Sales: Aug. 1, groceries, $5.60; Aug. 4, clothing, $7.80; Aug. 15, shoes, $6.25; Aug. 20, groceries, $3.85; Aug. 25, hardware, $5.60. Credits: Aug. 5, cash, $10; Aug. 15, services, $3.50; Aug. 28, cash, $15.

20. Willey & Son in account with A. P. Smith. Sales: Nov. 3, lumber, $18.60; nails, $1.20; paints, $3.50. Credits: Nov. 7, nails returned, $.50; cement returned, $1.20; paint returned, $.75; Nov. 20, cash, $18.

21. West & Son in account with E. R. Harris. Sales: $86.30; $94.30; $68.70; $42.30; $86.90; $75.80. Credits: By cash, $150; $75; $40; by returned goods, $16.50; $7.60. (Supply dates.)

22. A. Sellers & Co. in account with R. G. Lyons. Sales: \$168.80; \$90.30; \$84.70. Credits by returned goods, service, and cash: \$6.30; \$4.80; \$125; \$85. (Supply dates.)

3. BUYING AND SELLING AT A DISCOUNT

You, no doubt, have heard some one say that he bought or sold some article at a discount. A **discount** is a deduction from some former price. Thus, goods out of season or for cash often sell at a discount from a former or regular price.

1. If you should buy a bicycle listed at \$35 at a discount of 20 %, what would it cost you?

2. If a merchant gives a 5 % discount on all cash purchases, what is the yearly saving to a family that spends \$350 per year, regular price, at that store?

3. When a grocer advertises 10 ¢ package goods for 9 ¢, what per cent of discount is he giving?

4. When 25 ¢ packages are sold at 22 ¢, what is the per cent of discount?

5. When \$30 suits are selling at \$25, what per cent of discount is allowed?

6. When \$25 suits are selling at a discount of 10 %, how much will they cost?

7. A \$2500 automobile used for demonstration purposes was offered at a discount of 15 %. At this discount, how much will it cost?

8. A dealer advertised that he had an \$1800 automobile that had run less than 500 miles, which he would sell at \$1500. This was a discount of what per cent from the regular price?

9. A dealer during a "special sale" offers the following discounts:

A 10 % discount on all $350 parlor sets;
A 20 % discount on all $150 bedroom suites;
A 15 % discount on all $175 dining-room sets.

How much will a customer have to pay for each?

10. At a "special sale" a dealer offered the following prices:

All $35 suits, $30;
All $50 suits, $40;
All $25 suits, $16.50

Upon which class was the rate of discount the greatest? Can you give any reason for allowing different rates?

11. Find an advertisement of a "special sale" and reckon the rate of discounts allowed. Can you give a reason for the different discounts?

12. I bought an article for $21.60. The dealer told me that this was 25 % less than his former price. From this, show how to find the former price.

13. At a special "cash sale" I got a discount of 20 % from regular prices. At this "sale price" I bought the furniture for a new house at a total cost of $938.40. How much was saved over the former price?

14. At a "25 % discount sale," I bought goods costing me $70.95. What would they have cost me at the regular prices?

15. Make up problems in discount, giving what you consider a reasonable discount from former prices, giving as a reason that the goods were not in season, sold for cash, or any other reason for which you think a discount might be given.

4. COMMERCIAL OR TRADE DISCOUNT

The **wholesale merchant** who supplies the **retail merchant** with goods often has expensive catalogues of his goods, and in these catalogues he has a printed price called a **list price** from which he allows a discount to the dealers. The price that the goods cost the dealer after the discount has been deducted is called the **net price.** Since the discount is given by the wholesaler to the dealer handling his kind of goods, it is called a **trade** or **commercial discount.**

1. Athletic goods listed at $3.75 were sold to dealers at a discount of 30 %. Find the net price.

2. When golf shoes listed at $7.50 sold at $6.45 net, what per cent of discount was allowed?

From the following data, find the net price:

	List Price	Trade Discount		List Price	Trade Discount
3.	$38.50	30 %	8.	$ 96.80	10 %
4.	42.80	20 %	9.	142.50	30 %
5.	65.20	25 %	10.	84.30	40 %
6.	86.50	15 %	11.	72.40	20 %
7.	48.20	20 %	12.	64.70	15 %

From the following data, find the rate of discount:

	List Price	Net Price		List Price	Net Price
13.	$12.50	$10.00	18.	$13.50	$12.15
14.	16.80	15.12	19.	16.70	13.36
15.	26.25	21.00	20.	38.70	25.80
16.	42.80	29.96	21.	74.30	44.58
17.	36.50	25.55	22.	46.80	31.20

23. *Check the following bill:*

CHICAGO, ILL. *Aug. 4,* 1919

A. G. SPAULDING & CO.
ATHLETIC GOODS

SOLD TO Morgan & Dale
Dixon, Ill.

TERMS: *Net 30 days*

3	Drivers	$4.50	$13	50		
4	Mid irons	3.75	15	00		
2	Putters	3.25	6	50		
			35	00		
	Less 25 %		8	75	$26	25

5. SUCCESSIVE DISCOUNTS

Usually the list price of goods remains the same for long periods, but as the market changes, new discounts are made. When the market price decreases, it is usual for a new discount to be given and applied to the previous net price. Thus, if goods have sold for $15 less 20 % and the market goes lower, a further discount of, say, 10 % may be given on the former *net* price of $12. This is quoted as $15 less 20 % and 10 %.

1. How much will a dealer have to pay a wholesaler for goods listed at $84.50 less 20 % and 10 % ?

WORK

```
 5)$84.50
    16.90
10)67.60
     6.76
  $60.84
```

EXPLANATION.— A discount of 20 %, or $\frac{1}{5}$, is $16.90, leaving $67.60. A further discount of 10 %, or $\frac{1}{10}$, of this is $6.76, leaving $60.84, the *net price.*

2. Find the net price of goods listed at $86.40 less 15% and $12\frac{1}{2}$%.

WORK

$$\begin{array}{r} \$86.40 \\ .85 \\ \hline 43200 \\ 69120 \\ \hline 8)73.4400 \\ 9.18 \\ \hline \$64.26 \end{array}$$

EXPLANATION. — Since 15% is not an aliquot part of 100%, the first discount is deducted by finding 85% of $86.40, for if the discount is 15%, 85% of the list price remains. Since $12\frac{1}{2}$% is $\frac{1}{8}$, the last part is found as in problem 1, leaving a net price of $64.26.

3. Find the net price of goods listed at $94.50 less $33\frac{1}{3}$% and 10%.

From the following data, find the net price:

	List Price	Discounts		List Price	Discounts
4.	$27.80	25%, 10%	9.	$ 96.30	$33\frac{1}{3}$%, 10%
5.	36.50	20%, 10%	10.	120.50	15%, 10%
6.	54.90	$33\frac{1}{3}$%, 20%	11.	148.60	15%, 5%
7.	65.20	20%, 5%	12.	365.70	20%, 15%
8.	66.40	20%, $12\frac{1}{2}$%	13.	448.60	25%, 15%

14. *Check the following bill:*

BOSTON, MASS. *Oct. 13, 1919.*

SPENCER AND BROWN
SILVERWARE, CHINA, AND CUT GLASS

SOLD TO S. L. Scott & Son
Burlington, Vt.

TERMS: 60 da., 2% 10 da.

6 doz. plates	$3.20	19	20		
8 doz. dishes	6.80	54	40		
3 tea sets	4.25	12	75		
		86	35		
Less $33\frac{1}{3}$%		28	78	57	57
Less 10%				5	76
				51	81

15. What will the bill cost S. L. Scott & Son if paid before Oct. 23?

6. PROFIT AND LOSS

There are certain terms used in buying and selling that should be understood by all, for they are met in general reading and in conversation. These have to do with the profit or loss to one who sells goods.

The **prime or net cost** of an article is the amount actually paid for it. When transportation charges, insurance, commission for buying, etc., are added, the result is the **gross cost**. The **selling price** is what the dealer actually receives for the goods. The difference between the selling price and the gross cost is the **gross gain**, or **gross profit**. When all the expenses of selling, as salaries, traveling expenses, and all other costs of doing business, are deducted from the gross profit, the result is the **net profit**. In case the selling price is less than the gross cost, or if the cost of doing business is greater than the gross gain, there is a **loss**.

There is no uniform agreement among business men as to what should be used as the basis in finding the per cent of loss or gain. Some reckon it on the *prime cost*, some upon the *gross cost*, and some upon the *selling price*. No confusion arises, however, if the basis upon which it is reckoned is stated. But to say that a man made a profit of 25 % is meaningless unless the basis is stated, as "25 % of the prime cost," "25 % of the gross cost," or "25 % of the sales."

To ask what per cent a boy makes when buying *Saturday Evening Posts* at 3 ¢ and selling them at 5 ¢ is indefinite. He makes $66\frac{2}{3}$ % of the cost or 40 % of the selling price.

1. A boy sold brushes at $3, for which he paid $2.10. What per cent of the cost did he make? What per cent of the selling price did he make?

2. A dealer bought shoes at \$4.50 per pair. The cost of buying, delivery, etc., was 15 ¢ per pair. The cost of selling averaged 35 ¢ per pair. If the shoes sold for \$6.50, the net profit was what per cent of the prime cost? Of the gross cost? Of the selling price?

3. A retail grocer's sales for the year amounted to \$86,324.50. The gross profits were \$18,991.39. The entire cost of doing business was \$13,811.92. His gross profit was what per cent of the sales? His net profit was what per cent of the sales? The cost of doing business was what per cent of the sales?

4. If a wholesale grocer's sales for a year amounted to \$350,680 and he makes a gross profit of 12 % of the sales, and his expense of selling is 5.2 % of the sales, find the gross profit, the expense of selling, and the net profit.

5. If a dealer in hardware gets an invoice listed at \$387.50, less $33\frac{1}{3}$ % and 10 %, and sells it at a gross gain of 35 % of the net cost, how much does he get for it? If the cost of doing business is 22 % of the sales, find the net profit.

6. The "Profit and Loss Statement" of three departments of business one year showed the following: Clothing department, sales, \$94,500; gross gain, 32 % of the sales. Shoe department, sales, \$26,400; gross gain, 24 % of the sales. Men's furnishings department, sales, \$19,680; gross gain, 35 % of the sales. If the total cost of doing business averaged 20 % of the sales in each department, find the net gain in each.

7. If a merchant's sales for the year are \$83,450, with a gross gain of $23\frac{1}{2}$ % of the sales, what is his net profit if clerk hire is \$8640, and the other expenses, \$9364.50? The net profit is what per cent of the sales?

8. It is estimated that the average gross profit of the retail grocer is 21 % of his sales and that the gross profit of the wholesaler, of whom he buys, is 12 % of the wholesale price. In a city spending \$986,500 yearly for groceries, find the gross profit that goes to the retailer, and to the wholesaler.

9. An importer bought green coffee at 15 ¢, and sold it roasted at 24 ¢ per pound. If it lost 15 % of its weight in roasting, and the cost of selling was 2½ ¢ per pound, the net profit was what per cent of the selling price?

10. If a grocer pays \$1.25 per basket for peaches and sells them at \$1.75 per basket, after losing 10 % of them by decay, he is making a gross profit of what per cent of the sales?

7. COMMISSION AND BROKERAGE

One who buys or sells for others is often paid a per cent of the amount bought or sold. This fee is called his **commission** or **brokerage.** The one buying or selling is called a **broker** or a **commission merchant.** The general distinction between the two depends upon whether the agent actually handles the goods or not. If the one selling the goods actually handles them, he is usually called a commission merchant. If he merely arranges for the purchase or the sale, he is called a broker.

1. If a real estate agent sells a house for \$9500 and receives 2½ % of the sales as his fee, find the amount of the fee.

2. If a commission merchant sells \$2500 worth of produce on a 5 % commission, find the amount of his commission.

3. At 5 % find the commission of a shipment of 300 cases of eggs, 30 doz. per case, when sold at 43 ¢ per dozen.

4. If a broker sells \$236,500 worth of goods at $2\frac{1}{2}\%$, find the brokerage.

5. A truck gardener shipped his commission merchant vegetables which sold for \$238.50. After deducting 5% for commission and \$12.50 for freight and drayage, how much should the merchant remit to the gardener?

6. If an agent is selling goods on a 20% commission, how much per month will he make net from sales averaging \$1250 per month, after deducting \$95 per month for expenses?

7. An agent bought hogs for a shipper on a 2% commission, averaging \$18,500 worth per month, at an average expense of \$85. At this rate find his net earnings per year.

8. A boy sold aluminum ware on a 30% commission one summer vacation. His average sales per week were \$184.50 for 9 weeks, at a total expense of \$185. Find his net earnings.

9. A salesman in a large store got a fixed salary of \$1800 and 5% of the sales that he made. If his sales amounted to \$17,500 per year, find his total income.

10. If you know agents who work on commission, get data and information of interest and make a report of it to the class.

8. BORROWING AND LOANING MONEY

You often hear people speak of borrowing or loaning money, and of receiving or paying interest. **Interest** is money paid for the use of money or paid for an accommodation on an unpaid debt. It is reckoned as a certain per cent of the debt, called the **principal**, for a year's use of it, even though the interest is collected every half year or more often. The interest paid varies from 5% to 7%. Sometimes the

rate is even less than 5%, but seldom more than 7%. The rate of interest to be paid, unless paid in advance, is stated in the **promissory note** held by the debtor. This is a signed promise by the person borrowing the money or getting the accommodation that he will pay a certain sum of money at a specified time to the party holding the note.

USUAL FORM OF PROMISSORY NOTE

$400.00 New York, Apr. 5, 1919

Six months after date I promise to pay to

James F. Mason or order

four hundred and no/100 Dollars

for value received, interest at 6%.

Wm. D. Anderson

1. How much interest will be due on the above note at the end of six months?

2. Who holds the note? How much money will he receive when the note is due?

3. If your father loans $2000 at 6%, how much interest will he get each year? If the interest is payable every half year (semiannually), how much will he get in each payment?

4. If a man borrows $750 at 5%, how much interest will he have to pay each year?

5. A man bought a house for $12,500. Find the interest on the investment at $5\frac{1}{2}$%; that is, this amount loaned at $5\frac{1}{2}$% would earn how much per year?

Find the yearly interest of:

6. \$350 loaned at 6%.
7. \$940 loaned at 5%.
8. \$1150 loaned at $5\frac{1}{2}$%.
9. \$1580 loaned at 6%.
10. \$2450 loaned at 5%.
11. \$7500 loaned at 5%.
12. \$9600 loaned at $4\frac{1}{2}$%.
13. \$13,500 loaned at $5\frac{1}{2}$%.
14. \$16,250 loaned at 5%.
15. \$18,600 loaned at $4\frac{1}{2}$%.

CHAPTER XII

BANKING

You have seen banks in your city, been in them, no doubt, and perhaps you have money deposited in some savings bank. The savings bank will be discussed in the next chapter. The banks discussed here are **commercial banks.** They are institutions where money is deposited for safe keeping and paid out on the order of the depositor, and where drafts may be bought to send in payment of debt instead of sending the actual money. Banks also collect debts by means of drafts, and loan money on personal or other security. The modern bank is essential to the commercial life of the country and a great convenience to all of us.

DEPOSITED TO THE ACCOUNT OF
R. C. Lee
IN THE
FIRST NATIONAL BANK
OF DETROIT, MICHIGAN
May 10, 1919

	DOLLARS	CENTS
GOLD		
SILVER	9	40
BILLS	36	00
CHECKS	154	00
"	7	50
"	13	80
TOTAL	220	70

1. DEPOSIT SLIPS

When making a deposit with a bank you will fill out a **deposit slip** like the one in the margin and hand it, with the money and checks, to the **teller,** who checks each item. This serves as a sort of receipt of the transaction; for, in the future, if any questions arise as to the deposit, the slip in your own handwriting and checked by the teller is proof of the transaction.

Write out deposit slips for the following:

1. R. N. Doty deposited with the Farmers' Bank of Columbus, Ohio, on Apr. 3, 1919: $150 in gold, $380 in bills, and the following checks, $740, $36.80, $210.40, and $98.

2. C. L. Henry deposited in the Merchants' Bank of Kansas City, Mo., on Aug. 10, 1920: $285 in bills, $200 in gold, $56.50 in silver, and the following checks, $930, $178.50, $209.30, $16.80, and $18.32.

3. Make out a form of deposit slip for a "make-believe" bank of your school, as "The Students' Bank of the Detroit High School." Write out a deposit slip in which you are supposed to have deposited $10 in silver, $25 in gold, $45 in bills, and checks for $7.50, $8.35, and $16.80.

2. THE PASS BOOK

When making your first deposit with a bank, you will be given a **pass book** in which your deposit is entered to your credit, and in which future deposits will be entered. This pass book is left at the bank every month or so to be balanced. The following shows a page from such a book, showing the amount of each deposit and the total amount of the "vouchers returned." These **vouchers** are the returned checks that have been paid.

June 1, 1919	*balance*	*316*	*11*
5, 1919	*deposit*	*145*	*00*
7, 1919		*340*	*00*
9, 1919		*237*	*00*
18, 1919		*150*	*22*
30, 1919		*186*	*51*
	Total credits	*1374*	*84*
	Vouchers ret'd as per list	*876*	*52*
July 1, 1919	*balance*	*498*	*32*

The "vouchers ret'd as per list" item was the sum of all checks paid. This was found, perhaps, by an adding machine. The "list," shown in the margin, is returned with the balanced book and canceled checks.

21.04
20.57
2.16
42.11
33.50
17.28
38.88
60.65
1.44
55.61
41.17
66.25
29.80
40.00
115.00
27.71
21.74
60.81
41.96
68.60
5.45
29.00
6.45
20.84
8.50
876.52*

Rule forms like the above and balance the following bank accounts:

1. May 1, balance, $387.42. Deposited: May 5, $340; May 15, $635.70; May 25, $763.40. Vouchers returned, $1634.87.

2. June 1, balance, $496.34. Deposits: June 1, $842.60; June 10, $346.93; June 21, $684.75; June 30, $963.70. Vouchers returned, $2384.39.

3. Sept. 1, balance, $398.46. Deposits: Sept. 3, $348.90; Sept. 5, $196.30; Sept. 7, $206.30; Sept. 17, $587.65; Sept. 26, $498.76. Vouchers returned, $1598.32.

4. Dec. 1, balance, $286.70. Deposits: Dec. 2, $263.40; Dec. 10, $356.80; Dec. 13, $296.87; Dec. 28, $164.30; Dec. 30, $463.70. Vouchers returned, $1364.86.

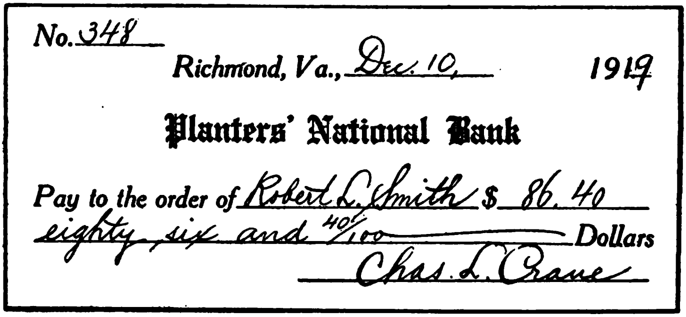
5. Oct. 1, balance, $489.40. Deposits: Oct. 5, $246.38; Oct. 7, $178.25; Oct. 10, $315.42; Oct. 16, $190.78; Oct. 24, $206.42. Vouchers returned, $1148.63.

6. Check the form at the head of this list. First see if the sum of vouchers returned is $876.52; then see if the pass book is properly balanced.

7. If you have a school bank, balance actual accounts. If not, make up and balance "make-believe" accounts.

3. MAKING OUT A CHECK

When you have a deposit with the bank you will be given a **check book** for writing out orders on the bank to pay out any of the money which you have on deposit. The usual form is shown below:



No. 348
Richmond, Va., Dec. 10, 1919
Planters' National Bank
Pay to the order of Robert L. Smith $ 86.40
eighty-six and 40/100 Dollars
Chas. L. Crane

1. Who is signing this order? Who, then, has money on deposit in the Planters' National Bank?

Before the bank will pay this money, Mr. Smith must **indorse** the check by writing his name, as it appears in the check, across the back of it.

The words "the order of" make the check **negotiable.** That is, Mr. Smith, by indorsement, may transfer it to some other person for collection instead of collecting it himself from the Planters' National Bank.

2. Suppose that R. L. Brown has a deposit in the Merchants' Bank of Indianapolis, Ind., and wishes this bank to pay $38.40 from the deposit to C. R. Reed. Write the proper form and show the indorsement.

3. Suppose that you have a deposit of $300 in the Students' Commercial Bank of your school. Write out a check to J. L. Hayes & Co. for $36.20 which you owe them.

4. Each check book has a form for entering frequent balances, new deposits, the amounts drawn, and for what purpose. The following is a common form:

DEPOSITS		AMOUNT	
BALANCE FORW'D		198	40
1919	March 4	250	00
TOTAL DEPOSITS		448	40
LESS CHECKS		89	43
BALANCE FORW'D		358	97

CHECKS DRAWN	AMOUNT	
NO. 426 DATE March 3, 1919	46	80
ORDER OF J. R. Smith		
FOR Feb. bill for groceries		
NO. 427 DATE March 3, 1919		
ORDER OF Public Service Co.	7	63
FOR Gas.		
NO. 428 DATE March 10, 1919		
ORDER OF Cash	35	00
FOR		
TOTAL CHECKS	89	43

5. Find the amount of one's credit when balance brought forward is \$296.80, and \$31.28 and \$92.24 had been deposited, and the checks drawn were \$126.40, \$84.70, and \$46.28.

Find the balance of credit from the following data:

6. Brought forward, \$196.87; deposits, \$34.96, \$78.27, and \$63.98. Checks drawn, \$84.37, \$27.68, and \$54.96.

7. Brought forward, \$208.76; deposits, \$84.26, \$154.37, \$75.80. Checks drawn, \$126.75, \$98.37, and \$42.96.

8. Brought forward, \$138.28; deposits, \$103.42, and \$116.28. Checks drawn, \$93.48, \$86.42, and \$74.39.

4. BUYING A DRAFT

If one wishes to pay a debt or send money to some one in another city, he may buy a **draft** of any bank and send that instead of the actual money. A draft is a written order from one bank to another bank to pay a specified sum to a third party. It is like a check, then, except that it is an order issued by a bank rather than by an individual.

USUAL FORM OF BANK DRAFT

No. 2468 **The Merchants' Bank**

Burlington, Vt., July 6, 1919

Pay to the order of E. B. Haynes & Co $ 246 $\frac{25}{}$

two hundred forty six and $\frac{25}{100}$ Dollars

To The National City Bank
New York

E. R. Porter
Cashier

1. At what bank is this draft bought?

2. Upon what bank is the order drawn?

3. Of what bank is Mr. Porter cashier?

4. Who is to receive the money?

5. What is the purpose of the words "to the order of"?

6. How can Haynes & Co. transfer this to some other party for collection?

7. Suppose that this draft was not bought by the party named in the draft (E. B. Haynes & Co.) but by J. C. Smith who wished to remit this sum to E. B. Haynes & Co. He could have had it made out to himself and then indorsed it over to E. B. Haynes & Co. by writing across the back, "Pay to E. B. Haynes & Co.," and then signing "J. C. Smith." Of the two methods, which would you think the better?

8. Suppose that E. L. Holmes wishes to remit to L. Harris & Bros. \$350, buying a draft of the First National Bank of Lansing, Mich., issued upon the Bankers' Trust Co. of New York. Fill out the two forms discussed above and show the indorsement of each.

9. Suppose that you wished to send $35 by draft to A. G. Spaulding & Co., Chicago, for athletic goods. Where would you get the draft, who would sign it, to whom would you have it made out?

5. BORROWING MONEY FROM A BANK

A bank's chief income is interest from the money that it loans. A large part of the money in any bank, which it loans, is that of its depositors. It is for the use of this money that banks can afford to take care of the money of their depositors and pay it out for them as they order, without making any charge for this service.

Banks usually loan their money to be paid "on demand" or for short periods, usually 30 da., 60 da., or 90 da. The interest on these time notes is paid in advance and is called **bank discount**, in distinction from *simple interest*, which is paid when the note is paid, or at fixed times.

USUAL FORM OF TIME NOTE

$300.00

Chicago, Ill., April 10, 1919

Sixty days after date I promise to pay to the order of the State Bank of Chicago

three hundred and no/100 Dollars

Payable at the State Bank of Chicago

Value received

No. 481 Due June 9, 1919 R. B. Morgan

No interest is named in the note, for it has been paid by Mr. Morgan at the time of the loan. If the bank's rate is 6%, it charged Mr. Morgan $3 interest (bank discount) at the time of the loan. This was taken from the $300 and $297 was given Mr. Morgan or credited to his account. The $297 is called the **proceeds** of the note.

Find the bank discount at 6% and the proceeds of:

1. $500 for 30 da.
2. $750 for 60 da.
3. $980 for 30 da.
4. $765 for 90 da.
5. $1250 for 30 da.
6. $1575 for 60 da.
7. $1500 for 90 da.
8. $1650 for 45 da.
9. $1860 for 30 da.
10. $1780 for 90 da.
11. $1560 for 70 da.
12. $1350 for 20 da.

For short periods (less than one year) 30 days are considered an interest month or $\frac{1}{12}$ of a year. Hence, at 6%, the interest is 1% for each 60 days. Thus, the interest of $1350 at 6% for 60 days can be seen at sight to be $13.50. For 30 days it would be half as much, or $6.75.

At sight give the discount at 6% of:

13. $1200 for 60 da.
14. $1950 for 60 da.
15. $2480 for 60 da.
16. $1375 for 60 da.
17. $1800 for 30 da.
18. $2400 for 30 da.
19. $1600 for 30 da.
20. $1450 for 30 da.
21. $1200 for 90 da.
22. $1600 for 90 da.
23. $2400 for 90 da.
24. $3600 for 90 da.

USUAL FORM OF A DEMAND NOTE

New York, March 7, 1919

On demand for value received, I promise to pay to the order of myself $1600.00

sixteen hundred and no/100 Dollars

with interest at 4%, at

The Market Exchange Bank
of New York

Chas. E. Meade

If the sum is large, or if a man's financial standing is not high, a bank will demand some **security**. This security will be one of two kinds. Either the note will be made out to the order of some one of high financial standing, who will indorse it and thus become responsible for the payment, or the borrower will put some security, called **collateral**, worth more than the face of the note, in the care of the bank to secure payment. This collateral will be sold by the bank to pay themselves if the note is not paid when due.

Often demand notes are given for large sums overnight or for a very few days.

Find the interest on the following demand notes of:

25. \$6000 for 5 da. at 4 %.
26. \$10,000 for 6 da. at $4\frac{1}{2}$ %.
27. \$12,000 for 1 da. at 5 %.
28. \$18,000 for 3 da. at 4 %.
29. \$36,000 for 8 da. at 5 %.
30. \$50,000 for 5 da. at 3 %.

6. DISCOUNTING NOTES AT A BANK

If one has a note and needs money before the note is due, he can discount it at a bank and get the money at once, the bank charging interest on the maturity value of the note for the time the note has yet to run. Or if banks themselves need more money, they may rediscount notes which they hold, at a Federal Reserve Bank.

Thus, if you have a note dated Apr. 4, 1919, for \$1200, interest 5%, to run 6 months, it is due Oct. 4, 1919, and worth \$1230 at that time. If you wish the money on this, Aug. 20, 1919, that is 45 days before it is due, a bank will buy the note, charging you interest at their regular rate on \$1230 for 45 days. At 6% this is \$9.23, and you will receive \$1230 – \$9.23 or \$1220.77, called the *proceeds*. That is, the solution is:

Face of note	\$1200
Int. for 6 mo. at 5%	30
Maturity value	\$1230
Discount of \$1230 at 6% for 45 da.	9.23
Proceeds	\$1220.77

1. Find the proceeds of a note of \$900, to run 8 mo. at 5%, discounted at 6%, 60 days before it is due.

2. If a merchant takes a 90-day note without interest for \$1200 for goods, and discounts it at 6% 20 days after date, how much will he get for it?

SUGGESTION. — Since the note does not bear interest, the maturity value is but \$1200. Being discounted 20 days after date, it has 70 days to run.

3. How much will a bank which charges 6% interest pay you for a note of \$750 to run 6 mo., bearing interest of 5%, if discounted 40 days before it is due?

4. Find the proceeds of a note of \$1500, dated May 5, 1919, to run 6 mo. at 6%, if discounted on Sept. 20, 1919, at 6%.

5. Find the proceeds of a note of \$1850, dated July 10, 1919, to run one year at 5½%, if discounted on May 16, 1920, at 6%.

6. A note of \$2400, dated Aug. 20, 1919, to run 8 mo. at 5%, was discounted at 6% on March 10, 1920. Find the proceeds.

7. A note of \$3600, dated Sept. 5, 1919, to run 4 mo. without interest, was discounted on Oct. 10, 1919, at 6%. Find the proceeds.

8. How much will a bank which charges 6% interest pay you for a note of \$1650, dated Nov. 15, 1919, to run 6 mo. at 5½%, if discounted on Feb. 10, 1920?

CHAPTER XIII

METHODS OF INVESTING MONEY

KNOWLEDGE of investments is a very fundamental part of one's education. At your age such knowledge is valuable to you in enabling you to understand and appreciate much that you read and the conversation that you hear in the home concerning investments. Later, when you have earnings to invest, such knowledge may prevent your being persuaded by the representatives of some "get-rich-quick" scheme to invest your money in some hazardous undertaking, and to enable you to invest more judiciously.

1. LOANING MONEY ON BOND AND MORTGAGE

The real standard by which the rate of income on an investment is measured is the rate at which money can be loaned on a note secured by a mortgage. For a number of years this has ranged from 5% to 6% of the investment (principal) per year. When an investment pays less than this, it has a *low* rate of income; when it pays more, it has a *high* rate of income

You have seen in Chapter XII that when money is loaned, the one receiving the loan gives his "promise to pay" or a promissory note together with some satisfactory security that the money will be repaid when due. The note is sometimes called a **bond** and the security given is sometimes a **mortgage**, which is an agreement that in case the one giving the note fails to pay the note or interest when due, certain

real estate or other property belonging to him may be sold to pay it. The mortgage becomes void when the money is paid. Thus, you hear one say that he has given a mortgage on certain property, which means that he has given his note secured by a mortgage. When one loans money on a note given to run for a long period of years and secured by a mortgage, he speaks of the transaction as loaning on "bond and mortgage." Under such a contract the interest is usually paid semiannually, or annually, the note and mortgage, however, running for several years.

1. At 5% what is the yearly interest on a note of $2500?

2. At 6% what is the semiannual interest on a note of $3500?

3. A man bought a home for $12,000, paying $7000 cash and giving a 6% mortgage on the home for the balance, interest payable semiannually. How much interest must he pay each half-year?

4. A man bought a farm for $18,000, paying half cash and giving a 5½% mortgage on the farm for the rest, interest payable annually. How much interest must he pay each year?

5. Mr. Taylor bought a house of Mr. Barnes for $12,500, paying $6,500 cash and giving him a 6% mortgage on the property for the rest, interest yearly.

(*a*) What is the face of the note?

(*b*) Who gives the note and who holds it?

(*c*) Who pays the interest? How much and when?

(*d*) What security has Mr. Barnes that he will get the interest when due, and the face of the note when due?

(*e*) Why would Mr. Barnes refuse to take a note for the whole value of the property secured by a mortgage on this property alone?

Find the yearly interest on:

6. \$9500 at 5%.
7. \$11,500 at $5\frac{1}{2}$%.
8. \$10,250 at 5%.
9. \$8750 at 6%.
10. \$4500 at 6%.
11. \$1250 at $5\frac{1}{2}$%.
12. \$1375 at 5%.
13. \$2125 at 6%.
14. \$1125 at $5\frac{1}{2}$%.
15. \$1450 at 6%.

Find the semiannual interest on:

16. \$9000 at 5%.
17. \$12,000 at $5\frac{1}{2}$%.
18. \$15,500 at 5%.
19. \$16,250 at 6%.
20. \$10,500 at 6%.
21. \$11,125 at 5%.

2. INVESTING IN BONDS

A **bond** is an agreement under seal to pay a certain sum of money at a stipulated time, with interest at a specified rate, issued by governments, municipalities, or corporations. In buying a bond, one should consider: (1) The safety of the principal; (2) The rate of interest paid; (3) The readiness with which it may be sold if he needs the money; and (4) The stability of its market value.

Government Bonds

You are all familiar with the *Liberty Loan Bonds* sold by the United States Government to help meet its expenses of carrying on the great World War. These were issued in denominations from \$50 to \$100,000. The Third and Fourth Liberty Loan Bonds paid $4\frac{1}{4}$% interest. In buying one of those bonds, you were merely loaning your money to your government and the bond which you held was the Government's promise to pay you the face of the bond at some specified time and to pay you a certain rate of interest

every half-year, until the bond was due. Over 20,000,000 people bought bonds in the Fourth Liberty Loan of over \$6,000,000,000. This was an average of a bond for nearly every family in the United States, and an average of nearly \$60 for each person.

States, too, issue bonds for internal improvements, as building roads, canals, bridges, schools, etc. These usually pay $3\frac{1}{2}\%$, 4%, or $4\frac{1}{2}\%$.

Government and state bonds are paid by a tax levied upon the people, and on account of this are considered the safest kind of investment. Hence, they find a ready sale at a rather low rate of interest.

There are two general types of bonds: the **coupon bonds** and the **registered bonds.** The *coupon bonds* have small coupons attached, which are certificates representing the interest due each period. As the interest becomes due, these may be cut off and deposited with a bank for collection. Thus, a \$1000 5% bond to run 20 years, interest payable semiannually, would have forty coupons attached, similar to the following:

A *registered bond* is registered in the name of the owner by the corporation issuing it, and a check is mailed to the owner as the interest falls due.

1. If you own a \$100 Liberty Bond paying $4\frac{1}{4}\%$ semiannually, how much is each coupon worth when due? Where can you get the money on the coupon?

2. What is the semiannual interest, at $4\frac{1}{4}\%$, on a \$500 Liberty Bond? On one for \$10,000? One for \$50,000?

3. What is the yearly interest on a New York State $4\frac{1}{2}\%$ bond of \$5000? On one for \$10,000?

4. How many \$1000 bonds bearing $4\frac{1}{2}\%$ interest will give an annual income of \$900? Of \$1890?

5. The Third and Fourth Liberty Loan Bonds bearing $4\frac{1}{4}\%$ interest amounted to \$10,782,980,000. What interest must the Government pay yearly on these two issues?

Municipal Bonds

Municipal bonds are those issued by cities, counties, and other political divisions of the state, and paid by special taxation. They generally run from twenty to fifty years and pay 4%, $4\frac{1}{2}\%$, or 5%. Occasionally the rate of interest is higher.

1. What is the difference in income between a government $4\frac{1}{4}\%$ bond for \$10,000, and a municipal bond of the same size for 5%? Which would you consider the safer investment?

2. If one holds a twenty-year $4\frac{1}{2}\%$ municipal bond of \$5000 from the time it was issued until it matures, what is the total amount of interest that will be received?

3. If a city issues \$1,000,000 worth of $4\frac{1}{2}\%$ bonds to build new schools, what yearly interest must it pay on the issue?

Railroad Bonds

Railroad bonds, as the name implies, are those issued by railroads. The security back of them is a mortgage on the company's property, as roadbeds, stations, terminals, equipment, etc. The safety of the security lies in the value of the property and the company's earning capacity. As these vary, the market value varies more than in the class of bonds already discussed.

Public Utility and Industrial Bonds

Public utility bonds are those issued by electric light, gas and power, street railway, and similar companies. **Industrial bonds** are those issued by manufacturing concerns, oil, coal, and steel companies, etc. These two classes of bonds, like the railroad bonds, are secured generally by a mortgage of the properties. As these depend upon trade conditions, their market value varies with the general state of the industry.

Yield or Investment Returns on Bonds

The **par value** of a bond is the *face value* or the sum named to be paid at maturity. The **market value** is the sum it can be bought or sold for in open market. The interest, as you have seen, is reckoned upon the par value. But the **yield** or **investment return** depends upon the price at which it was bought. Hence, if you pay "par" for a 5% bond, you get 5% on your money; if you pay *less* than par, you get *more* than 5% on your money; and if you pay *more* than par, you get *less* than 5% on your money.

Thus, if you pay $950 for a ten year $1000 bond paying 5% interest, you get $50 per year, and at the end of the ten years you get par value or $1000, thus making $50 *besides*

interest. This being an average of \$5 per year, you have really made \$55 per year on an investment of \$950, or about 5.79%. While if you paid \$1050 for it, you paid \$50 more than you get back at maturity, so there is a loss of \$5 per year, leaving a net income of but \$45 per year upon an investment of \$1050, or about 4.29%.

NOTE. — Since \$50 at the *end* of ten years is not the same as \$5 per year for ten years when interest is considered, this gives but an *approximate* yield or return. Any bond broker will give his customers the *exact* yield on bonds offered for sale.

1. Find the yield or investment return on a 5% bond for \$500, to run for 5 years, when bought for \$475.

2. Find the yield on the same bond if bought for \$510.

3. If a $4\frac{1}{2}$% \$1000 bond, due in 10 years, is selling for \$980, what is the yield?

4. When a \$1000 bond, due in 8 years, and paying 5% interest is selling at \$1020, what yield is that on the investment?

5. Find the yield on a \$1000, 3 year, 5% bond, when selling at \$970.

The Market Quotations

In the market reports found in the daily newspapers you will see such quotations as "N. Y. City $4\frac{1}{2}$'s, May '57 $98\frac{3}{4}$," "Un. Pac. 6's $103\frac{3}{8}$," etc. The first means that bonds issued by the city of New York, paying $4\frac{1}{2}$% interest, and due in 1957, are selling for $98\frac{3}{4}$% of their par or face value; that is, for $1\frac{1}{4}$% *below* par. The second means that bonds issued by the Union Pacific R.R., and bearing 6% interest, are selling for $103\frac{3}{8}$% of their par value; that is, for $3\frac{3}{8}$% *above* par. The first of these is sometimes said to be selling at a **discount**, and the other at a **premium**.

1. Find the cost of $4500 worth of bonds (par value) when selling at 102.

SOLUTION

$$\begin{array}{r} \$4500 \\ 1.02 \\ \hline 90\ 00 \\ 4500 \\ \hline \$4590.00 \end{array}$$

EXPLANATION. — The quotation means that the bonds are selling at 102 %, or 1.02 of their par value. Hence, for 1.02 × $4500.

2. Find the cost of a $5000 bond when selling at 98.

3. Suppose that a man bought eight $1000 bonds from the following quotation: "N. Y. Tel. $4\frac{1}{2}$'s $89\frac{1}{2}$." Find how much they would cost him and how much interest he would receive each year.

Bonds are usually bought and sold through an agent called a **bond broker**. His fee is usually $\frac{1}{8}$ of one per cent ($\frac{1}{8}$ %) of the face value of the bonds bought and sold. Thus, the fee for buying or selling a $1000 bond is $1.25. This fee is called **brokerage**.

4. Find the cost, including brokerage, of six $1000 bonds selling at 98.

SUGGESTION. — The total cost is $98\frac{1}{8}$ % of the par value.

5. How much will a man receive for five $1000 bonds sold through a broker at 98?

SUGGESTION. — After paying brokerage, he will receive but $97\frac{7}{8}$ % of the par value.

Find the cost including brokerage of:

6. $6000 at $88\frac{1}{2}$.

7. $7000 at $95\frac{7}{8}$.

8. $9000 at $101\frac{1}{2}$.

9. $6500 at $96\frac{3}{4}$.

Find the amount received for the following, if sold through a broker:

10. \$5000 sold at 98.

11. \$7500 sold at 101.

12. \$8500 sold at $89\frac{1}{2}$.

13. \$9500 sold at $87\frac{3}{4}$.

The price of a bond usually includes the statement "and accrued interest." This means that the buyer pays the interest that the bond has earned since the last coupon was due.

14. Find the cost of a \$1000 bond bearing 5% interest, payable Jan. 1 and July 1, quoted at $98\frac{1}{2}$, including brokerage, bought Apr. 1.

SOLUTION

\$1000 at $98\frac{1}{2}$	= \$985.
Int. from Jan. 1 to Apr. 1 @ 5%	= 12.50
Brokerage at $\frac{1}{8}$%	= 1.25
Total cost	= \$998.75

NOTE. — On July 1 the owner of the bond would cash his \$25 coupon reimbursing himself for the \$12.50 paid as accrued interest, leaving him \$12.50 as the interest from Apr. 1 to July 1.

15. Find the total cost, including accrued interest and brokerage, of a \$5000 bond paying $4\frac{1}{2}$% interest, payable Sept. 1 and March 1, if bought on Jan. 1 at $99\frac{1}{2}$.

Why Market Prices Change

You have seen that the interest earned on a "bond and mortgage" is the standard by which income returns on investments are regulated. Bonds are usually issued to run from twenty to fifty years. During that time money rates may change and this will cause a change in the price of the bond. Thus, if money on "bond and mortgage" is worth 6%, one would not pay par value for a bond paying but

4 %. On the other hand, if money is worth but 4 %, a bond paying 6 % would be worth more than par.

Another element that enters into the price of a bond is the security back of it. During such a long period the value of the property may change and thus affect the price of the bond. In general, the four leading factors that regulate the price of bonds are:

1. The security back of the bonds.

2. The rate of interest the bond is paying compared with the general interest rates of money.

3. The length of time the bond has to run

4. The confidence of the buying public in the stability and general earning power of the corporation issuing the bonds.

1. Would you expect a bond on a corporation heavily in debt and earning but little to sell above par or below par?

2. When general interest rates are 5 %, would you expect a 6 % bond on a prosperous corporation to sell for more or less than par?

3. When general interest rates are but 5 %, could you afford to pay 102 for a 6 % bond with good security if due in 2 years?

4. If the bond described in problem 3 had 5 years to run, could you afford to buy it at that price?

5. Describe a bond that you feel would not be worth par.

6. Describe one that you feel sure would sell for more than par.

3. SAVINGS BANK DEPOSITS

The investments with which you may be more acquainted are savings bank deposits. A **savings bank** is an institution for receiving and investing savings. Usually one cannot buy a bond or loan his money on "bond and mortgage"

unless he has $100 or more on hand. But he may start a savings bank account with $1. The accumulated deposits of a large number of depositors allows the bank an opportunity to invest these in bonds or loan them on mortgages at the usual rates of 5% or 6%, thus enabling them to pay the depositors $3\frac{1}{2}$% or 4%.

When the interest is due a depositor at a savings bank, it is not sent to him, but it is added to his account and thus begins to draw interest. When interest due is added to the principal and thus draws interest, the principal is said to be drawing **compound interest**, or the interest is said to be compounded. If the interest is added every six months, as in most savings banks, it is **compounded semiannually**; if it is added once a year, it is **compounded annually**.

1. If you deposit $50 on Jan. 2 in a bank, adding the interest on Jan. 1 and July 1 each year, how much will be added July 1, at 4%? How much will then draw interest until Jan. 1? How much interest will then be added?

2. If $500 is deposited on Jan. 2, 1920, in a bank paying 4% on Jan. 1 and July 1 each year, to how much will the principal and interest amount on July 1, 1924?

The amounts are reckoned more quickly by a table like the one on the following page.

By the tables find the amount of:

3. $500 at $3\frac{1}{2}$% for 20 yr.
4. $800 at 4% for 15 yr.
5. $300 at $3\frac{1}{2}$% for 20 yr.
6. $250 at 4% for 10 yr.
7. $375 at 4% for 15 yr.
8. $1200 at 3% for 5 yr.
9. $1500 at $3\frac{1}{2}$% for 10 yr.
10. $250 at 3% for 20 yr.
11. $750 at 4% for 10 yr.
12. $900 at 3% for 15 yr.

COMPOUND INTEREST TABLE

(The amount of one dollar principal)

YEARS	2 %	2½ %	3 %	3½ %	4 %	5 %	6 %	YEARS
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0500	1.0600	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.1025	1.1236	2
3	1.0612	1.0769	1.0927	1.1087	1.1248	1.1576	1.1910	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.2155	1.2625	4
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2763	1.3382	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3401	1.4185	6
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.4071	1.5036	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4775	1.5938	8
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.5513	1.6895	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.6289	1.7908	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.7103	1.8983	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.7969	2.0122	12
13	1.2936	1.3785	1.4685	1.5639	1.6651	1.8857	2.1329	13
14	1.3195	1.4130	1.5126	1.6187	1.7319	1.9800	2.2609	14
15	1.3459	1.4483	1.5580	1.6754	1.8009	2.0789	2.3966	15
16	1.3727	1.4845	1.6047	1.7340	1.8729	2.1829	2.5404	16
17	1.4002	1.5216	1.6529	1.7949	1.9479	2.2920	2.6928	17
18	1.4283	1.5597	1.7024	1.8575	2.0258	2.4066	2.8543	18
19	1.4568	1.5987	1.7535	1.9225	2.1069	2.5269	3.0256	19
20	1.4860	1.6386	1.8061	1.9898	2.1911	2.6533	3.2071	20
21	1.5156	1.6796	1.8603	2.0594	2.2788	2.7860	3.3996	21
22	1.5461	1.7216	1.9161	2.1315	2.3700	2.9253	3.6035	22
23	1.5770	1.7646	1.9736	2.2055	2.4647	3.0715	3.8198	23
24	1.6076	1.8087	2.0328	2.2835	2.5633	3.2251	4.0489	24
25	1.6405	1.8539	2.0938	2.3628	2.6658	3.3864	4.2919	25

4. THE GROWTH FROM REGULAR DEPOSITS

Of more interest to many are the amounts to which regular deposits will grow in a fixed time at compound interest. These are easily calculated by use of the following table:

TABLE SHOWING AMOUNT ACCUMULATED AT END OF A PERIOD OF YEARS BY PAYING $1 AT BEGINNING OF EACH YEAR IN THE PERIOD

Year	2 Per Cent	$2\frac{1}{2}$ Per Cent	3 Per Cent	$3\frac{1}{2}$ Per Cent	4 Per Cent	5 Per Cent	6 Per Cent	Year
1	1.020	1.025	1.030	1.035	1.040	1.050	1.060	**1**
2	2.060	2.076	2.091	2.106	2.122	2.152	2.184	**2**
3	3.122	3.153	3.184	3.215	3.246	3.310	3.375	**3**
4	4.204	4.256	4.309	4.362	4.416	4.526	4.736	**4**
5	5.308	5.388	5.468	5.550	5.633	5.802	5.975	**5**
6	6.434	6.547	6.662	6.779	6.898	7.142	7.394	**6**
7	7.583	7.736	7.892	8.052	8.214	8.549	8.897	**7**
8	8.755	8.955	9.159	9.368	9.583	10.027	10.491	**8**
9	9.950	10.203	10.464	10.731	11.006	11.578	12.181	**9**
10	11.169	11.483	11.808	12.142	12.486	13.207	13.972	**10**
11	12.412	12.796	13.192	13.602	14.026	14.917	15.870	**11**
12	13.680	14.140	14.618	15.113	15.627	16.713	17.882	**12**
13	14.974	15.519	16.086	16.677	17.292	18.599	20.015	**13**
14	16.293	16.932	17.599	18.296	19.024	20.579	22.276	**14**
15	17.639	18.380	19.157	19.971	20.825	22.657	24.673	**15**
16	19.012	19.865	20.762	21.705	22.698	24.840	27.213	**16**
17	20.412	21.386	22.414	23.500	24.645	27.132	29.906	**17**
18	21.841	22.946	24.117	25.357	26.671	29.539	32.760	**18**
19	23.297	24.545	25.870	27.280	28.778	32.066	35.786	**19**
20	24.783	26.183	27.676	29.269	30.969	34.719	38.993	**20**
21	26.299	27.863	29.537	31.329	33.248	37.505	42.392	**21**
22	27.845	29.584	31.453	33.460	35.618	40.430	45.996	**22**
23	29.422	31.349	33.426	35.667	38.083	43.502	49.816	**23**
24	31.030	33.158	35.459	37.950	40.646	46.727	53.865	**24**
25	32.671	35.012	37.553	40.313	43.312	50.113	58.156	**25**
26	34.344	36.912	39.710	42.759	46.084	53.669	62.706	**26**
27	36.051	38.860	41.931	45.291	48.968	57.403	67.528	**27**
28	37.792	40.856	44.219	47.911	51.966	61.323	72.640	**28**
29	39.568	42.903	46.575	50.623	55.085	65.439	78.058	**29**
30	41.379	45.000	49.003	53.429	58.328	69.761	83.802	**30**
31	43.227	47.150	51.503	56.334	61.701	74.299	89.890	**31**
32	45.112	49.354	54.078	59.341	65.210	79.064	96.343	**32**
33	47.034	51.613	56.730	62.453	68.858	84.067	103.184	**33**
34	48.994	53.928	59.462	65.674	72.652	89.320	110.435	**34**
35	50.994	56.301	62.276	69.008	76.598	94.836	118.121	**35**
36	53.034	58.734	65.174	72.458	80.702	100.628	126.268	**36**
37	55.115	61.227	68.159	76.029	84.970	106.710	134.904	**37**
38	57.237	63.783	71.234	79.725	89.409	113.095	144.058	**38**
39	59.402	66.403	74.401	83.550	94.026	119.800	153.762	**39**
40	61.610	69.088	77.663	87.510	98.827	126.840	164.048	**40**

1. If one can deposit $100 yearly in a savings bank paying 4 % yearly, how much will he have to his credit at the end of 5 yr.? Of 10 yr.? Of 15 yr.? Of 20 yr.?

2. If for 10 yr. a man can deposit $400 per year in a bank paying 4 %, how much will he have to his credit at the end of the 10th year?

3. If a man from the age of 30 to the age of 50 makes a regular yearly deposit of $300 in a savings bank paying $3\frac{1}{2}$ %, how much will he have in it by the end of that time?

To salaried people and others who can invest small sums monthly, a saving based upon monthly deposits is more interesting. The following table based upon 5 % is interesting. While savings banks seldom pay more than 4 %, after one's savings have grown to a few hundred dollars, he can easily make them earn 5 %. This table then is based upon the supposition that all interest is reinvested as it falls due.

APPROXIMATE GROWTH AND INVESTMENT RETURN BY MONTHLY PAYMENTS OF $10 — COMPUTED ON 5% BASIS COMPOUND INTEREST

Year	Capital at End of Year	Interest at End of Year	Year	Capital at End of Year	Interest at End of Year
1	123.27	6.16	16	2931.14	146.55
2	252.78	12.63	17	3202.80	160.14
3	388.85	19.44	18	3488.22	174.41
4	531.81	26.59	19	3788.08	189.40
5	682.01	34.10	20	4103.13	205.15
6	839.80	41.99	21	4434.12	221.70
7	1005.59	50.27	22	4781.87	239.09
8	1179.77	58.98	23	5147.22	257.36
9	1362.77	68.13	24	5531.07	276.55
10	1555.03	77.75	25	5934.35	296.71
11	1757.03	87.85	26	6358.05	317.90
12	1969.25	98.46	27	6803.20	340.16
13	2192.21	109.61	28	7270.89	363.54
14	2426.47	121.32	29	7762.25	388.11
15	2672.58	133.62	30	8248.74	413.92

From the table find the amount at 5 % of:

4. \$5 per month for 10 yr.
5. \$5 per month for 25 yr.
6. \$15 per month for 30 yr.
7. \$20 per month for 20 yr.
8. \$25 per month for 20 yr.
9. \$30 per month for 25 yr.
10. \$100 per month for 30 yr.
11. \$200 per month for 20 yr.

5. BUILDING AND LOAN ASSOCIATIONS

Many wage-earners and salaried people build homes through **building and loan associations.** These associations offer a safe form of monthly investment at a higher rate of interest than that paid by savings banks.

Such associations usually issue stock in \$100 **shares.** These shares are said to mature when the monthly **payments,** called **dues,** together with the **earnings** (largely interest on their loans), are equal to \$100. From the table on page 169, we see that if the net interest earned is 5 %, it will take \$1 monthly for about 7 years to mature a share of \$100.

If one borrows money from a building and loan association, he takes enough stock to cover the loan. His monthly payment must cover his monthly interest on the loan and pay the dues on the stock. When the stock matures, it cancels the indebtedness. The loan, of course, is secured by a mortgage on the property.

1. If one should borrow \$3000 to build a house, how many \$100 shares would he have to take out? At 50 ¢ per share, what would his monthly dues be?

2. What would the monthly interest at 5 % be on the loan of problem 1? What payment would have to be made monthly to the association?

3. If the earnings amount to 5 %, find by the table, page 169, about how long it would take to pay for the home.

SUGGESTION. — The table gives the accumulated amount of \$10. A monthly payment of 50 ¢ per share will give \$15, which will amount to $\frac{1}{2}$ more than \$10. Since $1\frac{1}{2} \times$ \$1969.25 is a little less than \$3000 and $1\frac{1}{2} \times$ \$2192.21 is a little more than \$3000, it will take between 12 and 13 years for the stock to mature.

4. Had the dues been \$1 per share, what would the total monthly payment have been?

5. At \$1 per share, about how long would it take for the stock to mature at 5 %?

6. If you have a building and loan association in your city, let a committee from the class find the amount of dues charged, the dividend it usually makes, the rate of interest charged for loans, the general time required for stock to mature, and any other points of interest, and make a report.

6. REAL ESTATE INVESTMENTS

In buying real estate, as houses, buildings, or farms to rent, one must consider not only the interest on the investment, but the taxes, repairs, insurance, and probable depreciation of value, in order to determine whether the investment pays.

1. If one pays \$10,000 for a house, what monthly rent must he get in order to make 6 % net on the investment, estimating that the expenses will average yearly: taxes, \$175; insurance, \$18.50; repairs and general upkeep, \$75; and general depreciation in value, \$150.

2. A man has a house which he can sell for $7500. He could loan this at 6 % on a note and mortgage. If he keeps it, he can rent it at $60 per month. The general yearly expenses are: taxes, $115; insurance, $13.50; repairs, $50. Not considering any increase or decrease in value, which will pay better and how much?

3. A man is offered a house which rents for $55 per month in exchange for six $1000 $4\frac{1}{2}$ % city bonds. If the general upkeep, taxes, etc., on the house will average $200 per year, what would you advise him to do?

4. In case the money returns on bonds and real estate are the same, which would you consider the better investment?

NOTE. — In answering such a question, there are many factors to be considered. Without knowing all these, no positive answer could be given. In general, bonds are more readily converted into cash in case one needs for other purposes the money invested.

7. INVESTING IN STOCKS

You have noticed from advertisements that most of the things you use are produced by some **company** or **corporation.** Thus, you see the advertisements of Swift & Co.; The Quaker Oats Co.; The National Biscuit Co.; Colgate & Co.; Cadillac Motor Car Co.; etc. These companies consist of a number of individuals united by the consent of the state, and empowered by the state to transact a certain form of business. The list of powers, rights, and duties of each are stated in writing in an instrument called their **charter.**

Stock is a name given the capital with which they do business. This **capital stock** is divided into **shares,** usually $100 each, called the **par value,** but they may be of any size. Thus, if a company has a capital of $1,500,000 divided into $100 shares, there will be 15,000 of them. Any one may

become a part owner of the company by buying one or more of these shares. If one owned 15 of these 15,000 shares, he would own one one-thousandth of the business and be entitled to that part of its earnings.

The owner of one or more shares of stock in a company is called a **stockholder** in the company. As evidence of ownership, each stockholder receives a **stock certificate** showing the number of shares he owns and the par value of each.

The earnings of a corporation that are divided among its stockholders are called the **dividends**. They are distributed as a per cent of the par value of the stock. Thus, a 12% dividend gives the holder \$12 for each \$100 share that he owns.

1. If the capital of a corporation is \$500,000 and divided into \$100 shares, how many shares will there be? The holder of 100 of these shares would own what part of the business?

2. If a man owns 100 of the 1000 shares in a company, he owns what part of the business? If \$8000 in earnings (dividends) are distributed, how much will he get?

3. If a company with a \$100,000 capital divides it into \$50 shares, how many shares will there be? For each share held, one will own what part of the business?

4. If a company with a capital of \$500,000 distributes \$75,000 in dividends, what per cent of the capital is this?

5. If one owns twenty \$100 shares in a company and an 8% dividend is declared, how much will he get?

6. If one owns twenty \$50 shares and a 10% dividend is declared, how much will he get?

7. When a company with a capital of \$1,000,000 declares a 6% dividend, how much will the whole dividend be? How much will a man get who owns fifteen \$100 shares?

8. When a man gets \$180 in dividends from fifteen \$100 shares of stock, what rate of dividend has been declared?

The Market Value of Stock

The **market value** of stock is the price at which it can be bought or sold in open market. A number of factors affect the market price of stock, chief among which are: (1) the real or prospective earning power of the corporation; and (2) the confidence of the buying public, or the lack of it, in the general stability of the enterprise. When the real or prospective earnings are small, the price is low; when large, the price is high.

As these two factors change with cost of labor and material, and the public demands for the company's products, and for numerous other reasons, the price of stock varies greatly. For that reason there is much speculation in stocks. By **speculation** is meant buying in expectation of a rise in price, or selling in expectation of lower prices, with the intention of buying back. In other words, speculation is dealing in uncertainties. A stock investment is always more speculative than a bond investment, owing to the fluctuation of the market value.

STOCKS AND BONDS COMPARED AS INVESTMENTS

Stocks	Bonds
1. The dividends depend upon the earning power of the corporation.	1. The interest is a fixed rate.
2. The dividends are not due until they have been declared by the board of directors.	2. The interest is paid at regular fixed periods.
3. Subject to sudden fluctuations in value.	3. Only slight fluctuations in value.

Stock quotations, like bond quotations, are a per cent of the par value. Thus, a quotation of "U. S. Rubber $70\frac{1}{4}$" means that United States Rubber Co. stock is selling for $70\frac{1}{4}\%$ of its par value, or \$70.25 for each \$100 share. Through a broker this would cost a buyer \$70.25 + \$.125 or \$70.375 per share and net a seller \$70.25 – \$.125 or \$70.125 per share. (The brokerage for buying and for selling is $\frac{1}{8}\%$ of the par value.)

1. Find the cost of twenty \$100 shares of U. S. Steel when quoted at $107\frac{1}{4}$, no brokerage. Find the cost of the same if bought through a broker.

2. How much will a man receive net from a sale of fifty \$100 shares of Studebaker Co. stock when quoted at $70\frac{1}{8}$, sold through a broker at $\frac{1}{8}\%$ brokerage?

3. How much is made on forty \$100 shares of Western Union Telegraph Co. stock when bought at $87\frac{1}{2}$ and sold at $92\frac{3}{8}$, after paying brokerage for both buying and selling?

4. When stock selling for $80\frac{1}{2}$ is paying a 6% dividend, what per cent of the investment is it earning? (The question is, "What per cent of \$80.50 is \$6?" Why?)

5. When stock selling for \$120 is paying an 8% dividend, what per cent does the investment yield?

Investing in Preferred Stock

The stocks already discussed, in which the stockholder becomes a part owner of the corporation by his investment, has a vote in the control of the business, and shares its profits, are called **common stocks.**

There is a growing tendency among industrial corporations to obtain capital requirements through the issue of a type of stock called **preferred stock.** This differs from the common stock in that the holder has no vote in the control of the

corporation, and does not share in the earnings except to the extent of the dividends guaranteed in the certificate. Preferred stock usually guarantees, in the certificate, a dividend of 7 %. It is sometimes but 6 %. Since preferred stock is not in the form of a note or bond against the company, secured by a mortgage on the property of the corporation, and having a date at which it matures, it is less safe as an investment and hence has to pay a higher rate in order to find buyers.

1. What is the yearly income from 40 shares of $100 each of 7 % preferred stock?

2. Find the difference in income between $10,000 invested in 7 % preferred stock at par, and the same amount invested in its $4\frac{1}{2}$ % bonds at par.

3. When "Liggett & Myers pf." is quoted at $107\frac{1}{8}$, what per cent of returns is the investor getting if the guaranteed dividend is 7 %?

4. A certain preferred stock paying a 6 % dividend is selling for 89; find the rate of income on the investment.

5. The total capital of a certain corporation is $2,700,000, of which $200,000 is 7 % preferred stock. One year the gross earnings were $296,465. The total expenses were $132,465. The net profits were all distributed as dividends. How much went to the holders of the preferred stock and what per cent of dividend did the holders of the common stock receive? In this case, which would you expect to sell higher in the market?

6. When common stock selling at 198 is paying a 12 % dividend and preferred stock selling at 105 is paying a 7 % dividend, which is giving the better return on the investment?

CHAPTER XIV

THE MEANING AND NATURE OF INSURANCE

You have heard people speak of **carrying insurance** on their property or on their health or life. **Insurance** is an agreement by an **insurance company**, for a consideration called a **premium**, to compensate the insured party for actual losses or damages arising from certain stipulated causes. The agreement or contract is called the **policy**. The sum of money specified in the policy to be paid in case of loss is called the **face of the policy**. There are two general classes of insurance: *property* insurance, and *personal* insurance.

1. PROPERTY INSURANCE

Property insurance, as its name implies, is insurance having to do with damage or loss of property of any kind. There are numerous kinds of such insurance, among the most common being fire, tornado, lightning, burglary, live stock, marine, plate glass, steam boiler, transit, automobile, etc. The principles of all are similar. The contract between the insured and insurer is always called the **policy**, and shows the conditions upon which the insurer agrees to indemnify for losses. As fire insurance is perhaps the most common of all these forms, it is discussed as representative of all.

Fire insurance is an agreement to compensate or indemnify the insured against *actual* losses arising from *accidental* fires. The "loss by fire" includes any damage resulting from chemicals or water used in extinguishing the fire. Fire caused by lightning is usually included under "accidental fires."

The **rate of premium** varies with conditions and is usually stated as a specified sum on each $100 in the face of the policy. The periods of fire insurance are usually one or three years.

1. For which do you think an insurance company would charge the larger premium: a $5000 policy on a house in a town with no fire protection, or the same amount on a house in a city having good protection?

2. For which would a company charge more in the same city: a policy on a modern fireproof building, or one of the same size on a wooden building with shingle roof?

3. Which do you think should pay the higher rate of premium on his policy: the owner of a home on his home, or the owner of a public garage on his garage?

4. Which would cost more, to insure a building in a neighborhood of brick or stone buildings, or the same building if it were surrounded by frame buildings?

The premium rate upon a building depends upon: (1) the location; (2) the nature of the construction; (3) the use made of the building; and (4) the construction and use of the adjoining buildings.

5. Some towns are divided into zones and the rates vary in each zone according to their distance from a fire department station. If you can, visit some local agent and get from him the rates where you live.

6. Rates are usually quoted as so many cents or dollars and cents per $100. 24 ¢ per $100 is what rate per cent?

7. If the rate on a 3-year policy in a certain city is $1.62 per $100, what is the rate per cent?

8. Show two ways of finding the premium on a 3-year policy of $6000 when the rate is 96 ¢ per $100.

9. For 15 years a man has kept his house insured at \$5000 by taking out 1-year policies at 48¢ per \$100. He could have taken out 3-year policies for $2\frac{1}{2}$ times the rate on a 1-year policy. How much could he have saved in taking out the longer policies?

10. Mr. Reed pays \$1.70 per \$100 on a \$3000 policy on a summer cottage in the country, and but 36¢ per \$100 on a \$5000 policy on his home in town. How much does he pay on each, and how do you account for the great difference in the rate?

11. A man has his property insured for \$2000 in one company and \$3000 in another. In case of a \$4000 loss, how much will he collect from each?

12. On account of the fire protection in most cities, a total loss of property by fire is unusual. For this reason a man often carries insurance to protect but partially the full value. Find the premium at 24¢ per \$100 on a policy covering but 80% of property valued at \$12,000. In case of total loss, how much would the policyholder receive? How much in case of a loss of \$6000? A loss of \$500?

13. In some states, if a man agrees, by accepting a certain clause in the policy, to carry a certain amount of insurance and fails to do so, he can collect (in case of loss) but such a part of it as the face of the policy bears to the amount agreed upon. Under such a contract, if a man agrees to carry \$5000 and carries but \$3000, what part of any loss up to \$3000 can he collect?

14. Some policies contain an 80% coinsurance clause, which is an agreement to carry 80% of the value of the property. How much insurance would a man have to carry under such a contract if his property is worth \$12,000?

15. A man having property worth $10,000 insures it for $6000. If there is an 80 % coinsurance clause in the contract, how much does he thus agree to carry?

16. If the man, problem 15, agrees to carry $8000 and carries but $6000, what part of a loss up to $6000 can he collect? (See problem 13.)

17. In 1916 the total premiums received by fire insurance companies were $419,361,346 and the total losses paid were $221,701,359. The losses were what per cent of the premiums? The expenses were $157,728,585. These were what per cent of the premiums?

18. In 1916 the policies written amounted to approximately $53,000,000,000. The losses paid were what per cent of this?

19. Can you bring to class a canceled fire insurance policy for study and discussion?

2. PERSONAL INSURANCE

Personal insurance is that form of insurance in which the insurance company agrees to pay a certain sum of money in case of accident to the insured, or in case of his sickness or death. These are called *accident*, *health*, and *life insurance*. Most common among these is life insurance.

There are four general forms of life insurance policies: (1) *ordinary life;* (2) *limited life;* (3) *endowment;* and (4) *term insurance.*

In the **ordinary life policy**, the premiums are paid, usually annually or semiannually, during the life of the insured, and the insurance company agrees to pay a fixed sum to the heirs of the insured, or to some other party designated in the policy, at his death.

The person named to receive this sum is called the **beneficiary.**

In the **limited life policy**, the premiums are paid for a fixed number of years, after which the policy is called **paid up**; but the face of the policy is not paid the beneficiary until the death of the insured.

In the **endowment policy**, the premiums are paid for a fixed number of years, as ten, fifteen, or twenty, and the face of the policy is paid the insured at the end of the period.

NOTE. — In both the limited life and the endowment, the face of the policy is paid the beneficiary in case of death before the end of the period during which premiums are payable.

In **term insurance**, the premiums are paid for a fixed period and the face of the policy is paid the beneficiary in case of death during this period. At the end of the period the contract ceases.

1. Would you expect an ordinary life policy or a twenty-payment life to cost more? Give a reason.

2. Would you expect a twenty-year endowment or a twenty-payment policy to cost more? Give a reason.

3. Arrange the four kinds of policies in order of what you consider the rate of premium.

THE TABLE SHOWS THE PREMIUM CHARGED BY A LEADING LIFE INSURANCE COMPANY FOR A $1000 POLICY. THE PROBLEMS THAT FOLLOW ARE BASED UPON THESE RATES.

AGE OF INSURED	ORDINARY LIFE	20-PAYMENT LIFE	20-YR. ENDOWMENT
20	$18.01	$27.82	$47.67
25	20.14	30.12	48.15
30	22.85	32.87	48.83
35	26.35	36.22	49.85
40	30.94	40.38	51.48
45	37.08	45.73	54.22
50	45.45	52.87	58.81

4. How much per year will a \$5000 ordinary life policy cost a man who insures at the age of 25? How much a year will it cost him if he insures at the age of 40?

5. Find how much a \$5000 policy of each of the three types will cost a man taking insurance at the age of 35.

6. Suppose that a man 30 years of age, taking out a \$15,000 policy, dies after making the 15th payment. His beneficiary would get \$15,000 under any of the three policies named above. Show how much he would have paid out in each.

7. If a man of 30 takes out a 20-year endowment policy of \$10,000 and lives 20 years, he will receive the face of the policy. How much less is this than the amount of the premiums if placed in a savings bank paying 4%?

8. If a man of 30 takes out an ordinary life policy of \$10,000 and dies in 20 years (after making 20 payments), would his beneficiary get more or less than the amount of the premiums if placed in a savings bank paying 4%?

9. Make the same kind of comparison as in problem 8, supposing that he died in 10 years.

10. Suppose a man of 50 should take a 20-payment life policy of \$20,000 and die at the end of 20 years. Compare what the beneficiary would receive with the amount of the premiums placed in a savings bank paying 4%.

11. Make the same kind of comparison as in problem 10, supposing the man to have been but 20 years of age when taking out the insurance and dying in 20 years.

12. Make up and solve other problems using the data from these tables.

The Three Elements that Make up the Premium

The **annual premium** paid by the insured is made up of three items: (1) **mortality cost**; (2) **reserve**; and (3) **expense loading.**

The *mortality cost* is the amount reckoned as necessary to collect each year to pay the death claims of that year. This is determined by "mortality tables" compiled from long experience, showing the deaths expected each year out of a certain number of any age.

The *reserve element* is the amount from each premium necessary to amount to the face of the policy in a given time. It is a sort of savings bank account of the insured with the company, bearing 3% or 3½% compound interest. It may be withdrawn at any time by surrendering the policy and thus terminating the contract, and is thus called the **cash surrender value** of the policy.

The *expense loading* is the amount estimated as necessary to meet the expenses of the management of the company. It is usually about one-fifth or one-sixth of the total premium.

1. A man 40 years of age taking out a $10,000 ordinary life policy at the rates given on page 181 may surrender it at any time after 2 years and get the reserve or "cash surrender value." At the end of 20 years, this cash value is $3834.70. This is how much less than he has paid out?

NOTE. — All rates refer to the table given on page 181.

2. If a man of 30 takes out an ordinary life policy of $1000, he may surrender it in 15 years and receive $276.02. This is how much more or less than he has paid out?

3. A man insuring for $1000 at 25 on the 20-payment plan may surrender it for $504.58 after having made the last payment. Compare this with the amount paid out.

4. A man insuring for \$10,000 at 30 on the 20-year endowment plan may surrender his policy in 15 years and receive \$6748.50. Compare this with what he has paid out.

5. The cash surrender value, at the end of 10 years, of a \$5000 20-payment policy taken by a man of 25 is \$1044.75. Compare this with the amount of the premiums at 4% compound interest.

CHAPTER XV

THE MEANING AND NECESSITY OF TAXES

Taxes are the money raised in some form to meet the expenses of government. These are raised in various ways to meet the expenses of the various units of government.

Towns and **cities** must raise money to meet the expenses of fire and police protection, of building and maintaining schools and other public buildings, to pay its officers, etc.

Townships and **counties** must meet the expenses of building roads and bridges, maintaining public institutions, paying certain salaries, certain courts, charities, etc.

The **state** has many salaried officials to pay, and helps build the roads of the state. It also keeps up certain state institutions, as prisons, schools, and asylums, all of which demands the expenditure of large sums of money.

The **United States Government** also requires large sums of money to meet its expenses. Among these are the salaries of its officials; the maintenance of its army and navy; interest on its national debt; and the pension of disabled soldiers. The total government expense of 1916 was about $725,000,000; during our first year in the great World War it rose to over $18,000,000,000.

1. HOW CITY, COUNTY, AND STATE EXPENSES ARE MET

Most of the expenses of towns, cities, counties, and states are met by a tax levied by the proper officers upon the **property** of the town, city, county, or state. The property

is divided into two classes for taxation: (1) **real estate**, regarded as immovable property, as lands and buildings, mines, railroads, etc.; and (2) **personal property**, including all movable property, as money, stocks, bonds, furniture, live stock, etc.

Assessors, elected or appointed, estimate the value of the property to be taxed. This is called the **assessed valuation** of the property. From the total assessed valuation and the tax to be raised, the **tax rate** is determined. This tax rate is stated in various ways. In some states it is a certain number of mills (tenths of a cent) on the dollar; in others, it is a certain number of dollars per \$100 or per \$1000. In some states, it is stated as a rate per cent.

Thus, a rate of $12\frac{1}{2}$ mills on the dollar is \$1.25 per \$100, \$12.50 per \$1000, or $1\frac{1}{4}\%$.

A FORM OF TAX BILL

RATES \$2.02

THIS BILL MUST BE RETURNED WHEN YOU PAY YOUR TAXES

Mr. *John Doe* — Page *123* Line *39*

No. *56 N. Walnut St.* — Map *3* Block *C* Lot No. *38*

Real Estate	Personal Property	Total Valuation	State, School, and County Tax	School Tax	Town Tax	Poll	Total Tax
7200	900	8100	55 64	35 08	72 90		163 62

1. If the assessed valuation of the property of a village is \$6,500,000 and \$97,500 is to be raised, the tax is how many mills on the dollar? How many dollars per \$100? What per cent?

2. If the school tax of a town is 4 mills on the dollar, how much school tax must a man pay whose property is assessed at $12,000?

3. When the town tax is $8\frac{1}{2}$ mills on the dollar, how much is the tax on property assessed at $9500?

4. When one's total tax is $1.98 per $100, what will he have to pay on property assessed at $17,256?

5. A man's total tax at $2.02 on $100 was $163.62. From this find at what value his property was assessed?

6. If taxes increase from $1.65 to $2.17 per $100, how much will it increase one's tax whose property is valued at $18,500?

7. Who is paying the highest rate, one who pays $10\frac{1}{2}$ mills on the dollar, $1.01 per $100, or 1.1%?

Give the rate per $100:

	Assessed Valuation	Tax to be Raised		Assessed Valuation	Tax to be Raised
8.	$4,800,000	$36,000	12.	$245,000,000	$1,250,000
9.	16,500,000	288,750	13.	356,000,000	1,850,000
10.	51,000,000	750,000	14.	758,000,000	3,762,000
11.	89,000,000	763,000	15.	986,000,000	10,248,000

Give the tax on:

	Assessed Valuation	Tax Rate		Assessed Valuation	Tax Rate
16.	$12,500	$4\frac{1}{2}$ mills on $1	20.	$6,780	$12.25 per $1000
17.	9,750	$9\frac{1}{2}$ mills on $1	21.	11,250	$17.60 per $1000
18.	10,500	$1.65 per $100	22.	17,750	$1\frac{3}{4}$ %
19.	13,750	$1.08 per $100	23.	16,350	$2.02 per $100

24. Find from the assessor of your city the assessed value of the property and the tax to be raised, and compute the tax rate.

2. HOW THE EXPENSES OF THE NATIONAL GOVERNMENT ARE MET

The people are not taxed directly upon the property they own, to support the National Government, as they are to support state, county, and local governments. The expenses are met chiefly by: (1) **tariffs, duties,** or **customs,** which are levied upon goods imported from other countries; (2) **internal revenue,** which is levied upon things made in this country, as alcoholic beverages and tobacco products; and (3) an **income tax,** levied upon the incomes of individuals and corporations.

Tariffs, Duties, or Customs

Some imported goods are not subject to duty. Such goods are said to be on the **free list.** The duties are of two kinds: (1) **ad valorem duty,** which is a per cent of the invoice price of goods at the place of purchase; and (2) **specific duty,** which is a certain amount per unit, as pound, ton, bushel, barrel, yard, etc. Some goods are subject to one duty and some to both.

The customs revenue is collected at **custom-houses** situated at the various ports of entry.

The tariff rates are frequently changed by Congress. For example, in 1913 the first income tax law was passed and the tariff rates were lowered that year.

1. The duty on watch and clock movements is 30 % ad valorem. Find the duty on a watch movement costing $8.50 in Europe.

2. The duty on drugs and medicines in pills, capsules, tablets, etc., is 25% ad valorem. Find the duty on an invoice valued at $16,528 in Europe.

3. The duty on olive oil in bottles and cans is 30¢ per gallon. Find the duty that an importer must pay on 15,000 gallons.

4. The duty on automobiles valued at more than $2000 is 45%. Find the duty on an automobile valued at $3500.

5. The duty on blankets and flannels is 30%. Find the duty on an invoice of $35,500 worth of flannels.

6. The duty on wool is 8%. In 1917 we imported 372,372,218 pounds, valued at $131,137,170. How much revenue did the government get from this one item?

7. In 1917 we imported $93,704,230 worth of manufactured copper ware at a duty of 20%. Find the revenue from copper.

8. Under the law of 1909, known as the Payne-Aldrich Tariff Law, the duty on wool was 30% ad valorem plus a specific duty of $24\frac{3}{4}$¢ per pound. Under the law of 1913, known as the Underwood-Simmons Tariff Law, the specific duty was dropped and the ad valorem duty lowered to 8%. In 1917 we imported 372,372,218 pounds, valued at $131,137,170. Find the decrease in revenue by the law of 1913 from that of 1909.

Internal Revenue

Before the World War we obtained nearly one half of the money needed to support the National Government from revenues on tobacco, spirits, and fermented liquors. In a recent year this internal revenue amounted to about $388,000,000.

1. During 1917 there were 60,729,509 barrels of fermented liquors (beer, ale, etc.) taxed at $3 per barrel. Find the amount of revenue from this source.

2. During the same year 7,390,183,170 cigars weighing more than 3 lb. per 1000 were taxed $4 per 1000. Find this tax.

3. At $2.05 per 1000, find the tax on 21,066,196,672 cigarettes sold in 1917.

4. At 13¢ per pound, find the tax on 417,235,928 pounds of chewing and smoking tobacco sold in 1917.

5. The following table shows the internal revenue receipts for 5 years. Make a bar graph for each item, showing the relative amounts each year.

Year	Spirits	Tobacco	Fermented Liquors
1913	$163,879,342	$76,789,424	$66,266,989
1914	159,098,177	79,986,639	67,081,512
1915	144,619,699	79,957,373	79,328,946
1916	158,682,439	88,063,947	88,771,103
1917	186,563,054	102,230,205	61,532,065

6. Make a bar graph of the internal revenues received in 1917, showing the relative amount received from each of three sources.

Revenue and Expenditures of the Post Office Department

The revenue from the sale of postage stamps is seldom listed under the internal revenues, for the income practically balances the expenses of the department. Thus, in 1917 the revenue of the department was $329,726,116 and the expenses were $319,838,718.

1. In 1917 the compensation paid to postmasters was $31,890,850. This was what per cent of the total expense

of $319,838,718? By inspection, estimate the result before solving and see how nearly correct you estimate.

2. The cost for transportation of mail in 1917 was $111,522,255. This was what per cent of the total expense? Estimate the result before solving.

3. In 1917 there were 43,338 rural delivery carriers, and the daily mileage was 1,112,556 miles. Find the average mileage per carrier.

4. The cost of rural delivery in 1917 was $52,420,000. That is what per cent of the total cost?

5. The cost of rural delivery increased from $41,859,422 in 1912 to $52,420,000 in 1917. Find the average increase per year.

6. From 1907 to 1917 the expenditures of the post office department increased from $190,238,288 to $319,838,718. Find the average yearly increase.

7. In 1917 the total cost of rural delivery was $52,420,000 and the total mileage was 1,112,556. Find the average yearly cost per mile for rural delivery.

8. There were 55,413 post offices in 1917, of which 10,381 were presidential appointment offices. This was what per cent of the whole?

The Income Tax

The income tax, as its name implies, is a tax upon incomes. This is a new form of raising money to support the government, having first been made a law in 1913. The rate has changed several times to meet new demands upon the government. The income tax is upon individuals and corporations. Since the beginning of this form of taxation the *personal* or *individual tax* has been divided into a **normal tax** and an **additional tax** or **surtax**.

The **normal tax** upon the incomes of 1918 was 12 % of the net income in excess of $2000 in case of a married person, and $1000 in case of an unmarried person, except upon the first $4000 of such excess, upon which the rate was but 6 %.

This law, which was passed in the early part of 1919, provided that the *normal tax* for each calendar year after 1918 should be 8 % of the net income (less the same exemptions as above), except upon the first $4000 of such excess, upon which the rate was to be 4 %.

The **surtax** provided in the law of 1919 was as follows:

Per Cent	From	To	Per Cent	From	To
1%	$ 5,000	$ 6,000	28%	$ 58,000	$ 60,000
2%	6,000	8,000	29%	60,000	62,000
3%	8,000	10,000	30%	62,000	64,000
4%	10,000	12,000	31%	64,000	66,000
5%	12,000	14,000	32%	66,000	68,000
6%	14,000	16,000	33%	68,000	70,000
7%	16,000	18,000	34%	70,000	72,000
8%	18,000	20,000	35%	72,000	74,000
9%	20,000	22,000	36%	74,000	76,000
10%	22,000	24,000	37%	76,000	78,000
11%	24,000	26,000	38%	78,000	80,000
12%	26,000	28,000	39%	80,000	82,000
13%	28,000	30,000	40%	82,000	84,000
14%	30,000	32,000	41%	84,000	86,000
15%	32,000	34,000	42%	86,000	88,000
16%	34,000	36,000	43%	88,000	90,000
17%	36,000	38,000	44%	90,000	92,000
18%	38,000	40,000	45%	92,000	94,000
19%	40,000	42,000	46%	94,000	96,000
20%	42,000	44,000	47%	96,000	98,000
21%	44,000	46,000	48%	98,000	100,000
22%	46,000	48,000	52%	100,000	150,000
23%	48,000	50,000	56%	150,000	200,000
24%	50,000	52,000	60%	200,000	300,000
25%	52,000	54,000	63%	300,000	500,000
26%	54,000	56,000	64%	500,000	1,000,000
27%	56,000	58,000	65%	over	1,000,000

How to Compute an Individual Income Tax

(For incomes of 1918)

Single Person: Net Income $7500

Income	$7500	
Exemption	1000	
Subject to normal tax	6500	
Tax on first $4000 of excess @ 6 %		240.00
Excess over $4000; $2500 @ 12 %		300.00
Surtax: Net income	$7500	
Not taxable	5000	
Subject to surtax	$2500	
From $5000 to $6000; $1000 @ 1 %		$10.00
From $6000 to $7500; $1500 @ 2 %		30.00
Total tax		$580.00

Head of Family — 3 Dependent Children: Income $15,000

NOTE. — Aside from the $2000 exemption, the head of a family is allowed $200 for each dependent child under 18 years of age.

Income	$15,000	
Specific exemption, $2000; plus allowance for children, $600		
Total allowance	2600	
Amount subject to normal tax . .	$12,400	
Normal tax, 6 % on first $4000 of excess of credits		$240.00
Normal tax of 12 % on taxable income over $4000; $8400 .		1008.00
Surtax: Net income	$15000	
Not taxable	5000	
Subject to Surtax	$10000	
Subject to surtax as follows:		
From $5000 to $6000 — $1000 @ 1 %		$10.00
From $6000 to $8000 — $2000 @ 2 %		40.00
From $8000 to $10000 — $2000 @ 3 %		60.00
From $10000 to $12000 — $2000 @ 4 %		80.00
From $12000 to $14000 — $2000 @ 5 %		100.00
From $14000 to $15000 — $1000 @ 6 %		60.00
Taxable income . . . $10000		
Total tax		$1598.00

Since the income tax rate changes frequently to meet the government's need of revenue, but few problems are given based upon the rates given here.

If interested in the subject, get the income tax rate at the time you study this and solve problems as your teacher may direct. The tax rate can be obtained at any bank.

1. A single man's income for 1918 was $6000. Find the income tax he had to pay in 1919. The head of a family would have had what tax upon the same income?

2. What was the income tax in 1919 for a single person whose income for 1918 was $10,000? A married person with 4 dependent children would have had what income tax upon the same income?

3. Find the income tax for the head of a family having 2 dependent children, if his income for 1918 was $8000.

CHAPTER XVI

SOME THINGS YOU HAVE LEARNED DURING THE YEAR

THIS chapter is a brief review of some of the new phases of the course learned early in the year. It may be used as a final review, or to supplement the topics as they are studied, or for both purposes.

1. YOU HAVE LEARNED TO INTERPRET AND EVALUATE A FORMULA

You learned to use letters for numbers in expressing a mathematical relation. These relations were called **formulæ** and you found that they were merely shorthand rules of computation.

*To **evaluate** a formula, you learned to substitute the numerical value of the letters and perform the computation.*

1. Evaluate $A = lw$ when $l = 12$ and $w = 8$. What principle of mensuration is expressed by this formula?

2. Evaluate $A = \frac{bh}{2}$ when $b = 16$ and $h = 12$. Interpret this formula as a principle in mensuration.

3. Evaluate $C = 2\pi r$ when $r = 20$. What principle of mensuration is expressed by this formula?

4. Evaluate $A = \pi r^2$ when $r = 24$. What principle of mensuration is expressed by this formula?

5. Evaluate $V = lwh$ when $l = 20$, $w = 15$, and $h = 12$, and tell what relation is expressed by the formula.

6. Evaluate $V = Bh$ when $B = 40$ and $h = 12$, and tell what relation is expressed by the formula.

7. Evaluate and interpret $V = \frac{Bh}{3}$ when $B = 60$ and $h = 15$.

8. Evaluate and interpret $V = \pi r^2 h$ when $r = 6$ and $h = 15$.

9. Evaluate and interpret $S = 4\pi r^2$ when $r = 8$. (The letters refer to a sphere.)

10. Evaluate $V = \frac{4}{3}\pi r^3$ when $r = 5$. Interpret the formula when the letters refer to a sphere.

11. What area is expressed by $A = \frac{1}{2}h(b + b')$? Evaluate the formula when $h = 8$, $b = 12$, and $b' = 10$.

12. The area of a triangle in terms of its sides is represented by the formula:

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

where A = area, S = half of the sum of the three sides, and a, b, and c are the length of the three sides. From the formula state a rule for finding the area of a triangle when its sides are known.

13. By use of the formula given in problem 12, find the area of a triangle whose sides are 20 in., 30 in., and 34 in., respectively.

14. How many acres in a triangular field whose sides are 24 rd., 30 rd., and 36 rd., respectively?

15. Evaluate $d(a - b)$ when $d = 10$, $a = 12$, and $b = 5$.

16. Find the value of $(x + y)^2$ when $x = 10$ and $y = 12$.

17. Find the value of $(r + 10) \div (7 - r)$ when $r = 5$.

18. If $S = a(t - \frac{1}{2})$, find S when $a = 40$ and $t = 8$.

19. Bring to class any formulæ you find in general reading or in other subjects that you are studying.

2. YOU HAVE LEARNED THE MEANING OF AN EQUATION AND HOW TO SOLVE IT

You have learned that an **equation** expresses the fact that one value equals or balances another; and that, just as in a scale pan, if any change is made on one side of the equation, the same change must be made on the other side.

You have learned, too, that one side of the equation contains an *unknown value* and that *to solve* the equation is to find a value for the unknown number that *satisfies* the equation.

Solve by inspection:

1. $x + 3 = 7.$	6. $2x = 10.$	11. $2x - 5 = 7.$
2. $4 + x = 9.$	7. $3x = 18.$	12. $\frac{1}{2}x = 5.$
3. $x + 7 = 12.$	8. $2x + 1 = 9.$	13. $\frac{1}{3}x = 6.$
4. $x - 3 = 5.$	9. $3x + 2 = 11.$	14. $\frac{1}{4}x = 3.$
5. $x - 7 = 10.$	10. $2x - 3 = 7.$	15. $\frac{1}{5}x = 4.$

16. Give the four axioms used in the solution of equations.

Solve and state the axiom used:

17. $n + 6 = 10.$	21. $3x - 10 = 20.$	25. $\frac{3}{4}n + 2 = 5.$
18. $3n = 2n + 8.$	22. $6x + 15 = 75.$	26. $8x - 10 = 70.$
19. $3n - 8 = 10.$	23. $3x - 7 = 29.$	27. $\frac{2}{5}x + 8 = 18.$
20. $5x + 2 = 27.$	24. $\frac{1}{4}n - 3 = 1.$	28. $\frac{2}{3}x - 7 = 3.$

3. PROBLEMS SOLVED BY USE OF EQUATIONS

You have learned to express a word-statement in the form of an equation and then to solve it. Further practice is here given.

1. A rectangular garden 60 ft. long contains 2400 sq. ft. Find its width.

Let $x =$ the number of feet in the width.
Then $60x = 2400$;
and $x = 40$, the number of feet in the width.

2. How wide a strip 40 rd. long will contain 3 acres?

3. Find the height of a triangle containing 68 sq. in. when the base is 12 in.

4. A rectangular prism 20 in. high contains 480 cu. in. How many square inches in the base?

5. Find the depth of a bin 8 ft. by 10 ft. that will contain 400 cu. ft.

6. There are 45 sq. ft. in the base of a pyramid containing 90 cu. ft. What is the altitude?

7. 20% of a certain number is 98. What is the number?

8. If 25% of a number is 120, what is the number?

9. James sold 75% of his pigeons and had 12 left. How many had he at first?

10. A merchant had forgotten the cost of an article, but remembered that he had marked it 25% above cost. If it was marked $30, what did it cost him?

11. At $4\frac{1}{4}$%, the interest on John's Liberty Bond is $4.25 each half year. He has a bond of what size?

12. In a Junior High School class in mathematics, there were 7 more boys than girls. In all there were 35. How many boys and how many girls in the class?

13. In another class of 36, there were twice as many boys as girls. How many of each?

14. One day Donald sold half as many papers as Ralph. Together they sold 90. How many did each sell?

15. It is 240 ft. around a rectangle 3 times as long as it is wide. Find the dimensions of the rectangle.

16. James caught 4 more than twice as many fish as Robert. Together they caught 34. How many did each catch?

17. Ralph and his sister raised vegetables to sell. They agreed that Ralph should do the heavy work and have twice as much of the money received as his sister. The total sales were $96. How shall they divide the money?

18. To make ice cream, Mary was going to use twice as much milk as cream. How much of each in 4.5 qt. of the mixture?

19. During a thrift-stamp campaign, Frank sold twice as many stamps as James did, and Ralph sold as many as both. Together they sold 240. How many were sold by each?

20. Together John and Ralph have 65 marbles. John has 5 more than Ralph. How many has each?

21. Ralph and Donald take care of Mr. Brown's lawn and garden for $75 for the summer. They agree that Ralph should have $1\frac{1}{2}$ times as much of the money as Donald. How much must each receive?

4. YOU HAVE LEARNED TO FIND DISTANCES BY SCALE DRAWINGS

1. Draw to scale 1 in. = 4 ft. a floor plan of a room 24 ft. by 32 ft. and by measurement find the diagonal of the room.

2. If the base of a triangle is 100 ft. and the base angles 50° and 60°, respectively, draw to scale 1 in. = 20 ft. a triangle and determine the other two sides by measuring the plan you have drawn.

3. Some boys wished to know the length of a small pond. They drove stakes at each end of the pond and found a point back from the pond from which they could measure to each stake. From this point it was 400 ft. to one stake and 500 ft. to the other, and the angle made by the two lines was 80°. Make a drawing to scale 1 in. = 40 ft. and find the length of the pond.

4. Some boys found the distance to a tree on the opposite side of a river from them by running a straight line 400 ft. long between two points, A and B, and noting the angles that the line of sight from each point made with line AB. If these angles were 60° and 80°, respectively, find how far the tree was from each point, by drawing a similar triangle to any scale.

5. A boy standing 80 ft. from the foot of a tree found that the angle of elevation to the top of the tree was 60°. By any scale you wish to use, find the height of the tree.

6. When a staff 8 ft. tall casts a shadow 10 ft. long, make a drawing to a scale and by the use of your protractor find the elevation of the sun.

7. From an observation balloon at an altitude of 6000 ft., the observer notes the enemy trenches are at an angle of depression of 20°. How far are the trenches from a point on the ground directly below the balloon?

NOTE. — The *angle of depression* is the angle made with the horizontal and is equal to the angle of elevation from the trenches to the balloon.

8. From a point 200 ft. above the surface of the water, the angle of depression of a boat is 15°. How far away is the boat?

5. YOU HAVE LEARNED TO FIND THE HEIGHT OF OBJECTS FROM THE LENGTH OF THE SHADOWS THEY CAST

You learned from a study of similar triangles that at any given time of day, the ratio of the shadow of an object to its height is constant, and hence any two such ratios form a proportion.

1. When a boy 5 ft. tall casts a shadow 8 ft. long, how high is a church tower that casts a shadow 240 ft. long?

FIRST SOLUTION

Let x = height of tower.

Then $\frac{x}{240} = \frac{5}{8}$;

and $x = 240 \times \frac{5}{8} = 150$.

Since the ratios are equal, they form a proportion. $\frac{x}{240}$ is the ratio of the height of the tower to its shadow, and $\frac{5}{8}$ is the ratio of the height of the boy to his shadow.

SECOND SOLUTION

$\frac{5}{8} \times 240 = 150$.

Since the boy's height is $\frac{5}{8}$ of his shadow, the height of the tower is but $\frac{5}{8}$ of its shadow.

2. When a staff 10 ft. high casts a shadow 8 ft. long, how tall is a tree that casts a shadow 120 ft. long?

3. Some boys found the distance across a stream by finding that a pole 20 ft. tall cast a shadow to the opposite bank when a rod 4 ft. tall cast a shadow 7 ft. long. Find the width of the stream.

4. When a flag pole known to be just 100 ft. tall casts a shadow 450 ft. long, how long a shadow will a boy 5 ft. tall cast?

5. An anchored observation balloon casts a shadow 1200 ft. from a point on the ground directly below it at the same time that a rod 5 ft. high casts a shadow 4 ft. long. How high is the balloon?

6. Measure heights in the vicinity of the school by use of the shadows that they cast.

6. YOU HAVE LEARNED TO FIND HEIGHTS AND DISTANCES BY TANGENT RELATIONS

You have learned that the *tangent* of an acute angle of a right triangle is the ratio of the side opposite the angle to the side adjacent to the angle.

1. From a point 100 ft. from the foot of a tree, the angle of elevation of the top of the tree is 50°. What is the height of the tree?

2. The base angles of an isosceles triangle are each 65° and the base is 30 ft. Find the altitude and area of the triangle.

SUGGESTION. — The altitude divides the triangle into two congruent right triangles.

3. When the angle of elevation from the top of a telephone pole is 40° at a point on level ground 60 ft. from the foot of the pole, what is its height?

4. An observer notes that the angle of elevation of an aëroplane is 60° when a second observer 2000 ft. away notes that he is directly below it. Find the height of the aëroplane.

5. When a flag pole 60 ft. high casts a shadow 80 ft. long, what is the elevation of the sun?

SUGGESTION. — The tangent of the angle is .75. To what angle does that most nearly correspond?

6. When the two legs of a right triangle are 50 in. and 60 in., respectively, what are the angles of the triangle?

7. When a balloonist whose altitude is 8000 ft. notes an enemy gun at an angle of depression of 25°, how far is the gun from a point on level ground directly below the balloonist?

8. From a ship, the angle of elevation of a light from a lighthouse known to be 80 ft. above the level of the ship is 8°. How far away is the lighthouse?

9. When the sun is 50° above the horizon, a church spire casts a shadow 65 ft. long. How high is the church spire?

7. YOU HAVE LEARNED TO REPRESENT DATA GRAPHICALLY

1. The average price received by the producer of butter over a range of six years was as follows: 1913, 28.4 ¢; 1914, 29.2 ¢; 1915, 28.7 ¢; 1916, 28.3 ¢; 1917, 34.0 ¢; 1918, 43.1 ¢. Show these relations by a bar graph. Also show the variation in price by a broken line or curve graph.

2. As in problem 1, show in both ways the variation in the price of eggs through a six-year period from the following data: 1913, 26.8 ¢; 1914, 30.7 ¢; 1915, 31.6 ¢; 1916, 30.6 ¢; 1917, 37.7 ¢; 1918, 46.3 ¢.

3. Show by graphs as above the variation in the price of farm land in Illinois during a four-year period when the prices per acre ranged as follows: 1915, \$110; 1916, \$115; 1917, \$120; 1918, \$132.

4. The following is the approximate population of the eight largest cities in the world. Show the relations by a bar graph: New York, 5,738,000; London, 4,523,000; Paris, 2,888,000; Tokio, 2,186,000; Chicago, 2,075,000; Berlin, 2,070,000; Vienna, 2,031,000; Petrograd, 1,900,000.

5. The following shows the per cent of our working population in the various occupations: agriculture, 33.2 %; mining, 2.5 %; manufacturing, 27.9 %; transportation, 6.9 %; trade, 9.5 %; public service, 1.2 %; professional service, 4.4 %; domestic service, 9.9 %; clerical occupations, 4.5 %. Show the relations by a bar graph.

6 The average weekly wages of factory workers for a five-year period were as follows: 1914, \$11.89; 1915, \$12.69; 1916, \$14.55; 1917, \$16.66; 1918, \$21.01. Show the variation by a broken line graph.

7. A man with an annual income of \$3000 used 75 % of it for living expenses, saved 20 % of it, and gave 5 % of it to charities. Show the distribution both by a circular graph and by a shaded bar graph. Which kind do you prefer and why?

8. In February, 1919, the price per pound of the best quality of sirloin steak varied as follows in the different sections of the country: San Francisco, 32¢; Seattle, 36¢; Denver, 36¢; Minneapolis, 28¢; Chicago, 37¢; Pittsburgh, 45¢; Philadelphia, 49¢; New York, 43¢; New Haven, 50¢; Boston, 56¢; and Portland, Me., 57¢. Show graphically the variation in price.

9. In January, 1919, the average sales of the War Savings Stamps were 45¢ for every person in the United States. The eight states leading in the sales that month were: Vermont, \$1.20 per capita; Montana, \$1.05; Utah, 94¢; North Carolina, 82¢; Idaho, 81¢; South Dakota, 75¢; Oregon, 72¢; and Colorado, 71¢. Make a graph by which these can be compared with the average sales in the United States and with each other.

8. YOU HAVE LEARNED THE USE OF MANY BUSINESS TERMS AND PROBLEMS

1. Write out a bill showing the amount due on the following purchases by Mrs. S. A. Smith of Howe & Co., Detroit, Mich.: Apr. 3, $5\frac{1}{2}$ yd. gingham at 48¢; $6\frac{3}{4}$ yd. satin at \$2.18; Apr. 12, 2 skirts at \$5.85; $\frac{7}{8}$ yd. ruffling at 36¢; Apr. 19, $2\frac{1}{2}$ yd. net at 85¢; Apr. 16, 1 skirt returned, \$5.85; Apr. 20, 2 waists at \$2.98; 3 pr. hose at 79¢.

2. Make out a bill from A. G. Spaulding & Co. (wholesalers), to E. L. Brown & Co. (retailers), Aug. 3, for: 5 doz. tennis rackets at $19.50 per dozen; 8 doz. tennis balls at $3.75 per dozen; 6 pr. athletic stockings at $1.15 per pair; 12 pr. tennis shoes at $2.95 per pair; 6 baseman's mitts at $4.20 each; and 4 Youth's League masks at $1.60 each. Allow discounts of 30% and 10%.

3. Write out an interest-bearing note covering a loan from E. R. Young to L. E. Barnes amounting to $950, to run 8 months at 6%, dated the day you study this problem. Find the interest. Who pays it and when? Who signs the note and who holds it? How much will the holder receive when the note is due?

4. Tell the ways that the payment, when due, might have been secured. That is, tell the kinds of security that might have been demanded or offered.

5. Write out a non-interest-bearing note such as E. L. Rice would be required to give The First National Bank of Topeka, Kansas, for a loan of $1200 for 90 days at 6%. How much interest would the bank get and when would it get the interest?

6. What is interest paid in advance called? What is the amount received by Mr. Rice for his $1200 note called?

7. Write out the form of a check given by A. M. Smoot on The Merchants' Bank for $12.75 to E. R. Holmes. Show how to indorse it and who endorses it. Where can Mr. Holmes get the money?

8. If you should want to purchase a draft of $18.75 to send to John Wanamaker & Co. for goods, tell where you could get it and show the form in which it should be made out.

9. If you have had experience in sending away for goods, tell how you transmitted the equivalent of money without actually mailing the money.

9. YOU HAVE LEARNED THE IMPORTANT METHODS OF INVESTMENT

1. What is meant by "loaning on bond and mortgage"? What rate of interest could you get in your community? Is this a safe kind of investment? (Discuss fully.)

2. In your community, what yearly interest would $2500 loaned on bond and mortgage yield? (To answer, you must know the rate of interest paid.)

3. What is a railroad bond? How is the bond secured? Are railroad bonds safe investments?

4. How much interest would the holder of a $5000 bond receive every half year if the rate is $4\frac{1}{2}\%$, payable semiannually?

5. What are the Liberty Loan and Victory Loan Bonds issued by the United States Government?

6. Find the semiannual interest on a $1500 Fourth Liberty Loan Bond paying $4\frac{1}{4}\%$.

7. When stock in some corporation is paying an 8% dividend, how much will the holder of ten $100 shares receive?

8. If one buys ten $100 shares of stock when quoted at 115, what will they cost without brokerage? With brokerage of $\frac{1}{8}\%$ of the par value?

9. What income from his investment will the holder of the ten shares (problem 8) receive when a 7% dividend is declared? Is this more or less than the interest which the

cost of the stock, including brokerage, would have earned at 6 %? How much?

10. If a man pays $12,000 for a house and rents it for $90 per month, is he making more or less than he would have made by loaning the money at 6 %, allowing $150 for taxes, $85 for repairs, and $200 for depreciation in value?

11. How many shares of stock could you buy for $1000 when quoted at 125, no brokerage? How much would you get in dividends if an 8 % dividend was declared?

12. Which would earn you the more money per year, stock bought under the conditions of problem 11, or a 6 % "bond and mortgage" for the $1000? Which would be the safer investment?

13. A man has $2000 to invest. He can buy 7 % preferred stock at par or loan his money on a mortgage at 6 %. Tell what you would advise him to do, and why.

14. If a man at the age of 25 is able to save $300 per year and continue this saving until he is 60 (35 years), keeping all interest reinvested at 5 %, find by the tables on page 168 how much he will have saved.

15. How much would the saving found in problem 14 earn yearly if loaned at 6 %?

16. The saving of $1 per week ($52 per year) for 20 years will amount to how much when placed in a savings bank paying 4 %? (Use the tables on page 168.)

17. If you had $5000 to invest, discuss the ways you could invest it, the probable returns from each, and the safety of each investment.

10. YOU HAVE LEARNED TO CHECK YOUR WORK AND TO KNOW THAT YOUR COMPUTATION IS CORRECT

Before the solution of a problem is of any value, we must *know* that the result is correct. This requires that every computation be carefully checked. The following exercises may be used as a final test of your skill in computation, or as drill-work throughout the term, or for both purposes.

Directions

1. *Write your name and the date on your exercise paper and be ready to begin work at a signal.*
2. *At a signal from your teacher, begin work on the exercise assigned.*
3. *Check each computation until you* **know** *that your results are correct.*
4. *Then hand in your work and your teacher will record the time taken.*

NOTE. — You may use these exercises for private drill when working for greater speed and accuracy. In that case, keep a record of the time taken for each exercise as you use them from time to time, then by comparison you can see what progress you are making.

Exercise 1

(*a*) Add 34.6, 9.47, 100.38, 96.475, 87.09, 432.8.
(*b*) From 300.98 subtract 96.09.
(*c*) Multiply 396.4 by 7.28.
(*d*) Divide 2534.916 by 73.2.

Exercise 2

(*a*) Add 500.4, 67.98, 175.9, 80.96, 8.175, 29.64.
(*b*) From 409.06 subtract 98.78.
(*c*) Multiply 93.42 by 6.29.
(*d*) Divide 3936.812 by 6.83.

Exercise 3

(*a*) Add 96.308, 207.96, 18.053, 203.9, 98.45, 72.623.
(*b*) From 720.06 subtract 196.8.
(*c*) Multiply 576.3 by 92.6.
(*d*) Divide 4868.916 by 76.7.

Exercise 4

(*a*) Add 278.16, 74.382, 97.65, 208.75, 96.84, 78.09.
(*b*) From 603.98 subtract 390.462.
(*c*) Multiply 82.46 by 37.8.
(*d*) Divide 3040.704 by 5.76.

Exercise 5

(*a*) Add 59.086, 403.97, 175.86, 93.42, 80.76, 175.9.
(*b*) From 480.93 subtract 198.47.
(*c*) Multiply 936.4 by 27.8.
(*d*) Divide 40,483.68 by 87.4.

Exercise 6

(*a*) Add 48.2, 305.95, 87.46, 130.95, 72.95, 204.8.
(*b*) From 601.28 subtract 97.375.
(*c*) Multiply 809.6 by 87.5.
(*d*) Divide 46,863.12 by 81.7.

Exercise 7

(*a*) Add 54.65, 108.38, 96.4, 308.75, 87.246, 30.49.
(*b*) From 98.026 subtract 49.36.
(*c*) Multiply 760.98 by 34.8.
(*d*) Divide 4926.396 by 52.7.

Exercise 8

(*a*) Add 304.9, 58.47, 89.42, 390.82, 19.43, 109.46.
(*b*) From 300.4 subtract 68.293.
(*c*) Multiply 576.8 by 9.37.
(*d*) Divide 2307.954 by 2.89.

Exercise 9

(*a*) Add 93.08, 46.93, 75.068, 138.9, 65.385, 73.9.
(*b*) From 560.92 subtract 178.39.
(*c*) Multiply 76.38 by 94.2.
(*d*) Divide 1991.672 by 3.76.

Exercise 10

(*a*) Add 40.68, 97.9, 240.75, 19.284, 63.9, 78.46.
(*b*) From 906.8 subtract 342.96.
(*c*) Multiply 893.7 by 4.58.
(*d*) Divide 3665.436 by 5.82.

Exercise 11

(*a*) Add $34\frac{1}{2}$, $56\frac{5}{8}$, $18\frac{3}{4}$, $16\frac{7}{8}$, $19\frac{9}{16}$, $24\frac{5}{16}$.
(*b*) From $342\frac{3}{16}$ subtract $195\frac{3}{4}$.
(*c*) Multiply 348 by $48\frac{3}{4}$.
(*d*) Divide $5384\frac{2}{3}$ by 7.

Exercise 12

(*a*) Add $28\frac{1}{3}$, $19\frac{5}{6}$, $14\frac{7}{9}$, $43\frac{1}{2}$, $32\frac{1}{6}$, $14\frac{5}{18}$.
(*b*) From $1508\frac{2}{3}$ subtract $509\frac{4}{5}$.
(*c*) Multiply 386 by $39\frac{4}{5}$.
(*d*) Divide $6039\frac{3}{4}$ by 8.

Exercise 13

(*a*) Add $42\frac{1}{2}$, $46\frac{3}{4}$, $18\frac{5}{6}$, $27\frac{2}{3}$, $17\frac{1}{4}$, $46\frac{7}{12}$.
(*b*) From $3048\frac{3}{8}$ subtract $932\frac{2}{3}$.
(*c*) Multiply 495 by $58\frac{2}{3}$.
(*d*) Divide $1730\frac{4}{5}$ by 6.

Exercise 14

(*a*) Add $20\frac{3}{4}$, $16\frac{5}{8}$, $43\frac{1}{2}$, $62\frac{5}{6}$, $13\frac{7}{12}$, $42\frac{2}{3}$.
(*b*) From $2061\frac{2}{5}$ subtract $973\frac{2}{3}$.
(*c*) Multiply 387 by $46\frac{5}{8}$.
(*d*) Divide $2631\frac{2}{5}$ by 8.

Exercise 15

(*a*) Add $34\frac{2}{3}$, $16\frac{5}{6}$, $21\frac{7}{8}$, $29\frac{2}{3}$, $46\frac{1}{2}$, $58\frac{3}{4}$.
(*b*) From $1706\frac{3}{8}$ subtract $950\frac{2}{3}$.
(*c*) Multiply 792 by $46\frac{3}{5}$.
(*d*) Divide $3842\frac{2}{3}$ by 7.

Exercise 16

(*a*) Add $43\frac{2}{3}$, $96\frac{7}{8}$, $45\frac{1}{2}$, $16\frac{3}{4}$, $17\frac{5}{12}$, $21\frac{1}{4}$.
(*b*) From $2096\frac{7}{16}$ subtract $906\frac{5}{8}$.
(*c*) Multiply 384 by $56\frac{4}{5}$.
(*d*) Divide $1763\frac{5}{8}$ by 5.

Exercise 17

(*a*) Add $53\frac{2}{3}$, $46\frac{7}{9}$, $18\frac{1}{2}$, $43\frac{5}{6}$, $14\frac{7}{18}$, $16\frac{2}{3}$.
(*b*) From $3105\frac{7}{8}$ subtract $1940\frac{2}{3}$.
(*c*) Multiply 564 by $57\frac{2}{3}$.
(*d*) Divide $2063\frac{5}{9}$ by 8.

Exercise 18

(*a*) Add $48\frac{1}{2}$, $19\frac{2}{3}$, $48\frac{5}{6}$, $15\frac{3}{4}$, $47\frac{1}{8}$, $54\frac{1}{3}$.
(*b*) From $1960\frac{5}{6}$ subtract $1068\frac{7}{8}$.
(*c*) Multiply 387 by $43\frac{7}{8}$.
(*d*) Divide $3046\frac{2}{3}$ by 8.

Exercise 19

(*a*) Add $52\frac{1}{3}$, $48\frac{3}{4}$, $16\frac{5}{8}$, $21\frac{5}{6}$, $12\frac{1}{2}$, $16\frac{3}{4}$.
(*b*) From $1930\frac{1}{2}$ subtract $398\frac{4}{5}$.
(*c*) Multiply 347 by $64\frac{4}{7}$.
(*d*) Divide $1296\frac{2}{3}$ by 7.

Exercise 20

(*a*) Add $32\frac{4}{5}$, $16\frac{2}{3}$, $34\frac{3}{5}$, $46\frac{4}{15}$, $17\frac{2}{3}$, $19\frac{4}{5}$.
(*b*) From $2063\frac{3}{8}$ subtract $970\frac{2}{3}$.
(*c*) Multiply 534 by $78\frac{2}{3}$.
(*d*) Divide $3576\frac{3}{4}$ by 7.

TABLES OF MEASURES

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 1 mile (mi.)
1 mile = 1760 yards = 5280 feet

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$272\frac{1}{4}$ square feet = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
1 square mile (sq. mi.) = 640 acres
1 acre = 43,560 square feet

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Liquid Measure

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
1 gallon = 231 cubic inches

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
1 bushel = 2150.42 cubic inches

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

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ANSWERS

JUNIOR HIGH SCHOOL MATHEMATICS

BOOK II

Page 2

1. 677,562.
2. 699,412.
3. 700,934.
4. 692,710.
5. 638,607.

Page 4

1. 224.328.
2. 173.551.
3. 230.361.
4. 228.462.
5. 232.823.

Fractions

1. $1\frac{3}{8}$.
2. $2\frac{1}{20}$.
3. $1\frac{11}{16}$.
4. $1\frac{1}{2}$.
5. $2\frac{1}{12}$.
6. $1\frac{5}{9}$.
10. $40\frac{1}{2}$.
11. $187\frac{5}{12}$.
12. $144\frac{1}{16}$.
13. $112\frac{7}{9}$.
14. $174\frac{11}{16}$.

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1. 36,783.
2. 52,609.
3. 32,806.
4. 11,386.
5. 16,827.
6. 33,459.
7. 17,728.
8. 11,296.
9. 32,784.
10. 3,275.
11. 35,947.
12. 27,334.
13. 26,617.
14. 11,804.
15. 4,577.
16. 18.953.
17. 38.82.
18. 22.546.
19. 22.295.
20. 1.885.
21. 18.665.
22. 22.285.
23. 3.195.
24. 27.316.
25. 3.475.
28. $20\frac{3}{8}$.
29. $26\frac{3}{8}$.
30. $13\frac{7}{8}$.
31. $8\frac{1}{4}$.
32. $30\frac{3}{4}$.
33. $67\frac{5}{6}$.
34. $57\frac{5}{6}$.
35. $42\frac{1}{6}$.
36. $136\frac{7}{8}$.
37. $72\frac{7}{8}$.

Page 8

1. 144,875.
2. 51,460.8.
3. 19,835.2.
4. 849.75.
5. 24.288.
6. 27,481.8.
7. 317.768.
8. 64,944.4.
9. 38,467.8.
10. 3,395.86.
11. $2\frac{2}{5}$.
12. $3\frac{3}{4}$.
13. $4\frac{2}{3}$.
14. $4\frac{4}{5}$.
15. $7\frac{7}{8}$.
16. $1\frac{7}{9}$.
17. $7\frac{1}{7}$.
18. $7\frac{1}{3}$.
19. $7\frac{7}{9}$.
20. $10\frac{2}{3}$.
21. $7\frac{3}{5}$.

Page 9

1. $\frac{9}{13}$.
2. $\frac{7}{15}$.
3. $\frac{14}{17}$.
4. $\frac{7}{12}$.
5. $\frac{8}{11}$.
6. $\frac{3}{4}$.
7. $\frac{2}{3}$.
8. $\frac{9}{16}$.
9. $\frac{2}{3}$.
10. $\frac{4}{5}$.
11. $\frac{3}{5}$.
12. $\frac{7}{8}$.
13. $\frac{16}{25}$.
14. $\frac{2}{3}$.
15. $\frac{5}{6}$.
16. $\frac{2}{3}$.
17. $\frac{15}{17}$.
18. $\frac{4}{7}$.
19. $\frac{4}{7}$.
20. $\frac{1}{2}$.
21. 64.575.
22. 56.235.
23. 227.695.
24. 47.025.
25. 783.58.
26. 1736.19.
27. 2732.29.
28. 428.582.
29. 353.976.
30. 628.56.

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21. 127.68.
22. 13.552.
23. 57.75.
24. 38.07.
25. 165.6.
26. 105.6.
27. 2.8536.
28. 15.6612.

29. 9.0615.
30. 212.
31. 48.112.
32. 287.04.
33. 816.
34. 608.4.
35. 986.1.
36. 3115.
37. 69,939.
38. 34,387.5.

Page 11

1. 2,400.
2. 12,000.
3. 3,500.
4. 48,000.
5. 12,000.
6. 72,000.
7. 48,000.
8. 63,000.
9. 4,500.
10. 6,400.
11. 7,800.
12. 8,400.
13. 12,800.
14. 63,000.
15. 54,000.
16. 8,000.
17. 6,000.
18. 9,000.
19. 16,000.
20. 10,800.
22. 612,000.
23. 2,268,000.
24. 1,344,000.
25. 292,400.
26. 2,450,000.
27. 2,726,400.
28. 3,444,000.
29. 2,856,000.
30. 1,752,600.

Page 12

1. 4,230.
2. 2,340.
3. 2,430.
4. 8,750.
5. 3,350.
6. 5,500.
7. 43,250.
8. 23,300.
9. 10,800.
10. 37,650.
11. 21,875.
12. 11,650.
13. 8,266$\frac{2}{3}$.
14. 12,750.
15. 7,466$\frac{2}{3}$.
16. 19,200.
17. 10,750.
18. 6858$\frac{1}{3}$.

Special Per Cents

1. 42.
2. 30.
3. 12.
4. 210.
5. 21.
6. 6.
7. 32.
8. 21.
9. 30.
10. 40.
11. 70.
12. 375.

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1. 135,408.
2. 237,800.
3. 368,064.
4. 284,172.
5. 457,866.
6. 336,336.
7. 379,134.
8. 283,392.
9. 530,712.
10. 196,664.
11. 272,808.
12. 703,248.
13. 396,396.
14. 378,432.
15. 275,415.
16. 300,330.
17. 149,420.
18. 240,798.

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1. 295.4166.
2. 678.3809.
3. 934.8865.
4. 12,054.054.
5. 1,172.8531.
6. 35.5081.
7. 54.7159.
8. 42.4416.
9. 20.4489.
10. 8.3811.
11. 2.4567.
12. 63.1746.
13. 173.6734.
14. 347.5675.
15. 165.3571.
16. $\frac{9}{10}$.
17. $\frac{14}{15}$.
18. 2$\frac{1}{12}$.
19. 1$\frac{5}{16}$.
20. 1$\frac{1}{5}$.
21. 2$\frac{7}{24}$.
22. 1$\frac{5}{9}$.
23. 1$\frac{1}{14}$.
24. 1$\frac{5}{13}$.
26. 92$\frac{5}{12}$.
27. 82$\frac{8}{21}$.
28. 807$\frac{31}{32}$.
29. 185$\frac{1}{2}$.
30. 92$\frac{2}{3}$.
31. 74$\frac{19}{32}$.
32. 80$\frac{7}{12}$.
33. 116$\frac{7}{30}$.
34. 136$\frac{1}{15}$.
35. 66$\frac{23}{63}$.
36. 71$\frac{7}{13}$.
37. 96$\frac{1}{4}$.

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1. 23.425.
2. 22.458.
3. 1.959.
4. .9781.
5. .941.
6. 4.8485.
7. 1.753.
8. 2.461725.
9. 1.801.
10. 1.3888.
11. 1.1081.
12. .2133.
13. .2477.
14. .1765.
15. .14229.
16. .1158.
17. .0921.
18. .01936.
19. .028916.
20. .01185.
21. .0324.
22. .005325.
23. .008.
24. .008.

Dividing by Aliquot Parts

1. 153.84.
2. 50.91.
3. 49.36.
4. 130.26.

5. 85.26.
6. 78.72.
7. 171.6.
8. 147.68.
9. 50.94.
10. 173.2.
11. 69.78.
12. 81.84.
13. .2696.
14. 9.66.
15. 13.264.
16. 1.3008.
17. 2.2548.
18. 4.984.
19. 4.328.
20. 7.712.
21. 3.7072.

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1. 142.
2. 250.
3. 639.
4. 152.
5. 126.
6. 252.
7. 480.
8. 25.
9. 576.
10. 25.
11. $22\frac{1}{2}$.
12. 35.
13. 504.
14. 516.
15. 612.

Miscellaneous Drill

1. 10,800.
2. 56,000.
3. 72,000.
4. 4,210.
5. 2,420.
6. 2.450.
7. 21,000.
8. 17,400.
9. 4,600.
10. 18,800.
11. 7,600.
12. $4,666\frac{2}{3}$.
13. 104.
14. 142.
15. 112.
16. 133.
17. 154.
18. 143.
19. 125.
20. 179.
21. 127.
22. 105.
23. 142.
24. 137.
25. 48.
26. 18.
27. 18.
28. $37\frac{1}{2}$.
29. $9\frac{1}{2}$.
30. $3\frac{1}{3}$.
31. 8.
32. $19\frac{1}{3}$.
33. 27.
34. 12.
35. $10\frac{2}{3}$.
36. 28.

Pages 17–18

1. 30.
2. 66.
3. 452.3904 sq. ft.
4. 942.48 cu. in.
5. 942.48 cu. in.
6. $c = \pi d$.

Pages 18–19

1. $21\,a$.
2. $20\,c$.
3. $15\,m$.
4. $23\,x$.
5. $17\,y$.
7. 2480.
8. 7850.
9. 31.416.
10. 17,920.
11. 11,300.
12. 10,500.
13. 18,600.
14. 3,750.
15. 726.
16. 987.

Page 19

1. $7\,a$.
2. $5\,b$.
3. $8\,x$.
4. $7\,y$.
5. $5\,c$.
7. 2,750.
8. 1,600.
9. 1,920.
10. 2,250.
11. 9,750.
12. 9,820.
13. 3,200.
14. 4,100.
15. 2,160.
16. 3,800.

Pages 20–21

Multiplication

1. $12\,b^2$.
2. $18\,a^2$.
3. $21\,y^2$.
4. $36\,c^2$.
5. $32\,b^2$.
6. $56\,c^2$.
7. $63\,d^2$.
8. $40\,r^2$.
9. $30\,a^3$.
10. $40\,a^3$.
11. $80\,b^3$.
12. $80\,c^3$.
13. $30\,bc$; $18\,rs$; $30\,at$; $42\,mn$.
14. $35\,ab$.
15. $42\,bc$.
16. $54\,de$.
17. $56\,ac$.
18. $42\,bd$.
19. $54\,ad$.
20. $56\,de$.
21. $48\,cb$.
22. $72\,eg$.

Division

1. $2\,a$.
2. $3\,c$.
3. $8\,a$.
4. $3\,d$.
5. $8\,y$.
6. $9\,b$.
7. $8\,c$.
8. $8\,c$.
9. $8\,c^2$.
10. $7\,d^2$.
11. $7\,ab$.
12. $4\,a$.
13. $9\,d$.
14. $9\,z$.
15. $8\,b$.
17. 4.
18. 4.
19. 4.
20. 6.
21. 6.
22. 9.
23. 7.
24. 8.

25. 9.
26. 9.
27. 7.
28. 8.

Page 21

Factoring

1. $3(a+b)$.
2. $7(b+c)$.
3. $5(c+d)$.
4. $8(a+c)$.
5. $7(a+d)$.
6. $10(x+y)$.
7. $9(a+b)$.
8. $2(a^2+b^2)$.
9. $3(ab+cd)$.
10. $5(ab^2+c^2)$.
11. $6(ab+d)$.
12. $4(xy+ab)$.
13. 700.
14. 900.
15. 800.
16. 400.
17. 450.
18. 700.

Pages 22–23

2. $d=\frac{c}{\pi}$.
3. $B=\frac{V}{h}$; $h=\frac{V}{B}$.
4. 47.74 ft.
5. 30 sq. in.; 20 in.
6. $r=\frac{c}{2\pi}$.
7. $2A=h(b+b^1)$.
8. $b=\frac{2A}{h}$; $h=\frac{2A}{b}$.
9. 8 in.
10. $a=\frac{V}{bc}$.
11. $h=\frac{2A}{b+b^1}$.
12. 6 in.
13. 92.3 sq. ft.
14. 5.769 ft.
15. 7.503 ft.
16. 210.08 ft.
17. 8.53 in.
18. 68 ft.
19. 16 in.

Page 25

1. 6.
2. 4.
3. 15.
4. 11.
5. 5.
6. 5.
7. 6.
8. 9.
9. 18.
10. 25.
11. 7.
12. 14.
13. 8.
14. 9.
15. 17.

Page 26

1. 5.
2. 9.
3. 26.
4. 7.
5. 9.
6. 7.2.
7. 7.25.
8. $1\frac{3}{4}$.
9. 4.83.
10. 5.72.
11. 5.
12. $6\frac{1}{3}$.
13. 14.75.
14. 6.4.
15. 8.7.

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Division

1. 5.
2. 6.
3. 4.
4. 6.
5. 7.
6. 6.
7. 6.
8. 5.
9. 2.
10. 7.
11. 7.
12. 7.
13. 8.
14. 7.
15. 3.

Addition

1. 11.
2. 22.
3. 5.
4. 11.
5. 16.
6. 173.
7. 180.
8. 22.6.
9. 13.1.
10. 3.9.
11. 5.
12. 7.
13. 5.
14. 60.
15. 40.

Page 28

1. 16.
2. 27.
3. 35.
4. 48.
5. 54.
6. 7.
7. 9.6.
8. 20.
9. 23.8.
10. 10.8.
11. $7\frac{1}{2}$.
12. $2\frac{2}{3}$.
13. $5\frac{5}{6}$.
14. $5\frac{1}{3}$.
15. 6.

Miscellaneous

1. 4.5.
2. 17.
3. 11.
4. 6.
5. $4\frac{1}{2}$.
6. 12.
7. $4\frac{2}{3}$.
8. 9.
9. 6.5.
10. 9.
11. 5.4.
12. 2.2.
13. $7\frac{1}{2}$.
14. $7\frac{1}{2}$.
15. 9.
16. $6\frac{3}{4}$.
17. 8.
18. $\frac{21}{32}$.

Pages 29–30

2. 6 in.; 10 in.
3. 14 boys; 18 girls.
4. 50 ft.; 70 ft.
5. 16 ft.; 20 ft.

6. 58 ; 59.
7. 40; 41; 42.
8. 27 ; 29.
9. 48 ; 50.
10. 43 ; 55.
11. 30¢ ; 60¢.
12. 15 rd.; 30 rd.
13. 14 ; 24.
14. 80 ft.; 120 ft.
15. $5 ; $10.
16. 20.
17. 2¢ ; 4¢.

Pages 31–32

2. $\frac{1}{3}$; $\frac{1}{2}$; $1\frac{1}{3}$; $1\frac{1}{2}$.
3. $\frac{2}{3}$.
4. $\frac{3}{5}$.
5. $1\frac{1}{2}$.
6. $1\frac{1}{3}$.
7. 5.
8. $1\frac{1}{4}$.
9. $\frac{4}{7}$.
10. 4.
11. 2.2588+.
12. .804+.
13. .5368+.
14. .574+.
15. $\frac{3}{4}$.
16. $\frac{12}{13}$.
17. 90 ft. ; $53\frac{1}{3}$ ft.
18. $1\frac{3}{7}$; 7 in.
20. $\frac{2}{3}$.

Pages 34–36

2. $\frac{4}{9}$.
3. $\frac{ab}{a^1b^1}$.
5. $\frac{a}{c}$.
7. $\frac{b}{c}$.
8. $\frac{ab}{a'b'}$.
9. $\frac{5}{9}$.
10. $\frac{ab}{a'b'}$.
12. $\frac{7}{9}$.
13. $\frac{a}{c}$.
14. $\frac{25}{49}$.
15. $\frac{a^2}{a'^2}$.
19. 4 times.
20. 9 times.
21. 6.
22. $1\frac{1}{2}$.
23. 4.
24. $\frac{1}{16}$ as great.
25. $10.

Pages 37–39

8. 80°.
10. 20 ft.

Pages 39–43

1. $12\frac{1}{2}$ ft.
2. 36 ft.
3. 35 ft.
4. 65 ft.
6. $\frac{AB}{AE}=\frac{DC}{EC}$.
7. 300 ft.
8. $12\frac{1}{2}$ ft.
9. 380 ft.
10. 190 ft.
11. $27\frac{7}{9}$ ft.
13. $266\frac{2}{3}$ ft.

Pages 44–45

2. $366\frac{2}{3}$ mi.
3. 855 mi.
4. 19′ 6″ by 9′ 6″ ; 8′ 6″ by 9′ ; 18′ 6″ by 6′.
6. 1 in. = 4 ft.
7. 2″ by 5″.
8. 3″ by 4″.
9. 4″ by 7″.
10. 672 ft.
11. 120 ft.
12. 1 in. = 20 ft.
13. 40′ by 8′9″; 8′ $1\frac{1}{2}$″ by 6′ 3″ ; 40′ by 27′ 6″.

Page 47

4. 69.6 ft.
5. 82.5 ft.
6. 59.5 ft.

Pages 48–50

1. 61.44 ft.
2. 588 ft.
3. 1027.5 ft.
4. 553.19 ft.
5. 571.42 ft.
7. 31°.
8. 3300 ft.
9. 66°, 24°.
10. 15.277 ft.
11. 5405.4 ft.
12. 140.74 ft.

Pages 52–59

2. Hart 2.2 times.
4. About 30 % less.
6. About 10 %; 32% ; 65 %.
7. About 83 %; 118 %.
8. About 2 % ; 22 % ; 50 %.
9. About 45 % ; 85 % ; 105 %.
10. About 13 %; 15 % ; 25 %.
11. About 64.88 %.
12. About 52 %.
13. About 5 % increase.
14. About 18 % increase.
17. 160 – 14 % of Gt. Br.; 204.33 % of France ; 335.28 % of Italy.
18. 46.67 % ; 143.11 %.
19. 141.86 % ; $143\frac{1}{3}$ %; $50\frac{2}{3}$ %.
20. 20 %.
21. 49.18 %.
22. About 45 % ; 48 %.
25. 70.74 %.
26. 18.42 % ; 19.81 % ; 8.47 %.
28. Cattle 56 millions ; Sheep 52 millions ; Hogs 62 millions.
29. Australia 87.91 % ; Germany $11\frac{1}{9}$ %.
30. Germany 48.14 % ; Australia .8 %.

31. About 32.94 %; 50.59 %; 36.47 %.

32. $22\frac{2}{9}$ %; $77\frac{7}{9}$ %.

33. Cattle 39.47 %; Sheep 42.1 %; Hogs 18.42 %.

34. *a.* $-41\frac{2}{3}$ %; 40.47 %; $17\frac{6}{7}$ %;
b. 48.66 %; 37.95 %; 13.38 %;
c. 122.32 % of France;
d. $142\frac{6}{7}$ % of France.

Pages 60–63

1. About $2\frac{1}{2}$; $2\frac{8}{9}$; $2\frac{2}{3}$.

2. 150 %; $188\frac{8}{9}$ %; $166\frac{2}{3}$ %.

3. 258.28 %; 299.28 %; 277.11 %;

4. 1.95 in.; 4.48 in.; 4.19 in.

Pages 64–67

1. 61° 30′; 275° 30′.

2. Food \$450; Rent \$300; O. Ex. \$$283\frac{1}{3}$; Clothing \$250; Misc. \$$216\frac{2}{3}$.

3. 2481 millions; 2337.5 millions.

5. Beef 51.88 %; 50 % (graph).

6. *a.* Debt 5.9 % of wealth;
b. Debt of 1918 403.16 % of Debt;
c. 15.03 % of Gt. Br; 20.71 % of France; 91.79 % of Italy.
d. 112.85 % of Gt. Br.; 142.7 % of France; 388. 62 % of Italy;
e. Debt 44.31 % of wealth;
f. Debt 40.67 % of wealth;
g. Debt 24.99 % of wealth.

Pages 68–70

1. 1880 – about 4.28 times 1850; 1890 – 6 times 1840; 1880 – 44.11 % of 1910.

5. *a.* 100 %;
b. 10 %;
c. 11 %;
d. 30 %;
e. 10 %.

Pages 77–80

2. 67 qt.
3. 259 pk.
4. 56 ft.
5. 948 min.
6. 2296 sec.
7. 25 pt.
8. 2016 sq. rd.
9. 4576 rd.
10. 10,030 yd.
11. 267 oz.
12. 804 sq. in.
13. 151 cu. ft.
15. 4 gal. 2 qt.
16. 62 qt. 1 pt.
17. 16 ft. 4 in.
18 9 yd. 16 in.
19. 2 A. 106 sq. rd.
20. 20 lb. 4 oz.
21. 43 bu. 3 pk.
22. 61 yd. 2 ft.
23. 5 hr. 42 min.
25. 9 in.
26. 2 ft.
27. 3 qt. 1 pt.
28. 10 in.
29. 14 oz.
30. 115 sq. rd.
31. 3 pk. 6 qt.
32. 1 ft. 8 in.
33. 16 min.
34. 100 sq. rd.
35. 625 lb.
36. 56 sec.
37. 10 in.
38. 12 oz.
39. 3 qt.
40. 3 qt. 1 pt.
42. .208 hr.
43. $\frac{7}{9}$ yd.
44. $\frac{7}{8}$ gal.
45. $\frac{3}{11}$ mi.
46. .409 mi.
47. .6875 bu.
48. .408 da.
49. .611 yd.
50. .6667 sq. ft.
51. 9 in.
52. 18 in.
53. 22 posts.
54. 8; 21.

Pages 80–81

2. A = Iw.
3. $I = \frac{A}{w}$; $w = \frac{A}{I}$.
5. \$86.40.
6. \$35.
7. 15,930 sq. ft.
8. 2312 sq. ft; 880 sq. ft.
9. 63.71 %.

Pages 84–85

3. 265 lb.
4. 93 lb.

Page 86

3. 48 sq. in.
6. 480 sq. in.

Pages 87–88

12. 60°.
13. 50°.
14. 30°; 60°.

Pages 89–90

3. $27\frac{1}{2}$ sq. in.
4. $14\frac{5}{8}$ A.
5. $44\frac{1}{16}$ A.

Pages 90-92

2. 37.6992 ft.
3. 68.661 ft.
4. 105 ft.
5. 8.3776 ft.
6. .5236 ft.
7. $\frac{1}{15}$; .5236 ft.
8. Add $\frac{1}{32}$ of it.
9. 31.416 in.
10. 18.8496 ft.
11. 47.124 ft.
12. 4.7746 ft. 2.3873 ft.
13. 1.5915 in.
14. 1.9099 in.

Pages 92-93

2. 452.3904 sq. ft.
3. 113.0976 sq. ft.
4. 392.7 sq. ft.
5. $\frac{4}{9}$ as large.
6. 4 times; $1\frac{7}{9}$ times.

Pages 93-95

1. 14 bd. ft. 14 boards.
2. 6 bd. ft.
3. 18 bd. ft.
4. 36 bd. ft.
5. $21\frac{1}{3}$ bd. ft.
6. $20.16.
7. $30.88.
8. $2\frac{1}{4}$ in.
9. $\frac{1}{3}$.
10. 400 bd. ft.; 600 bd. ft.; 2000 bd. ft.
11. $36.

Pages 95-97

1. 5 ft. by 3 ft. by 2 ft.
2. 30 cubes.
3. 15 cubes; 30 cubes.
4. 60 cu. in.
5. $V = abc$.
7. 480 cu. ft.; 13.71 T.
8. 1 sq. in.; 1 sq. ft.
9. 15 sq. units.
10. 20 cu. ft.
11. 120 cu. ft.
12. $V = Bh$.
14. 249.33 gal.
15. 576 bu.
16. 675 cu. ft.
17. 240,000 cu. ft.

Pages 97-98

1. 6.8544 gal.
2. 31.9872 gal.
3. 423.0144 gal.
4. 1884.96 cu. in.
5. 783.36 gal.
6. 112.5949 tons.

Page 98

3. 47.124 sq. ft.
4. 43.9824 sq. ft.

Pages 99-100

2. 60 cu. ft.
3. 282.744 cu. ft.
4. 32 cu. ft.
5. 34.906 loads.
6. 53.6166 bu.
7. 180.95616 bu.
8. 15.708 bu.

Pages 100-102

3. 314.16 sq. in.; 804.2496 sq. ft.
4. 201,062,400 sq. mi.
6. 523.6 cu. ft.
7. 2144.6656 cu. in.
8. 523.6 cu. in.
9. 32.0705 lb.
10. 1.9584 gal.
11. 3.8083 tons.
12. 66.5408 lb.
13. $\frac{1}{8}$ as large.
14. $\frac{1}{27}$ as large.
15. 64 times.
16. 2.37 times.
17. 125 times.

Page 104

1. 3969.
2. 5184.
3. 7225.
4. 2209.
5. 1444.
6. 9216.
7. 3249.
8. 1225.
9. 8649.
10. 7056.
11. 1849.
12. 8281.
13. 5776.
14. 2809.
15. 7569.
16. 7921.

Pages 105-106

1. 28.
2. 58.
3. 92.
4. 56.
5. 83.
6. 52.
7. 73.
8. 67.
9. 99.
10. 87.
11. 54.
12. 97.
14. 532.
15. 547.
16. 636.
17. 746.
18. 869.
19. 2453.
20. 728.
21. 696.
22. 799.
23. 852.
24. 543.
25. 1319.
26. .25; .1225; .060025.
29. .75.
30. .96.
31. 6.498+.
32. .885−.
33. .943+.
34. 4.412+.
35. 28.721.
36. .8.
37. .253−.
38. 43.959−.
39. 15.03.
40. .894+.
41. 1.414+.
42. 1.732+.
43. 2.236+.
44. 2.645+.
45. 3.16+.
46. 4.24+.

47. 4.898+.
48. 6.245−.

Page 107

By First Method

1. 67.99.
2. 76.03.
3. 93.52.
4. 43.87.
5. 38.44.
6. 58.84.
7. 30.62.
8. 26.86.
9. 25.16.
10. 28.56.
11. 27.15.
12. 31.17.
13. 8.73.
14. 9.197.
15. 6.83.
16. 8.69.
17. 5.93.
18. 5.36.
19. .62.
20. .76.
21. .97.
22. .92.
23. .87.
24. .71.

Page 109

1. 73 sq. in.
2. 134 in. by 67 in.
3. 23.44 ft.
4. 17.32 ft.
5. 22.627 ft.
6. 25.98 in.
7. 60.39 ft.
8. 81.05 ft.

Pages 111–112

1. 60 ft.
2. 68 ft.
3. 57 ft.
4. 115 ft.
5. 42.426 ft.
6. 25.61 in.
7. 21.63 mi.
8. 223.6 rd.
9. 19 ft. (18.97 ft.)
10. $33\frac{1}{3}$ yd.
11. 127.27 ft.
12. 207.385 ft.
13. .3535 in.
14. 11.66 in.
15. 13.75 in.
16. 68.22 sq.in.
17. 173.205 sq. in.

Page 114

1. .45.
2. .63.
3. .07.
4. .09.
5. 1.38
6. .16.
7. .165.
8. .04.
9. .045.
10. 2.4.
11. .154.
12. .036.
13. .008.
14. 1.254.
15. 2.456.
16. .045.
17. .0625.
18. .0875.
19. .095.
20. 2.
21. 156.
22. 37.43.
23. 48.64.
24. 4.394.
25. 167.7.
26. 82.175.
27. 1225.
28. 1560.
29. 3220.
30. 1198.75.
31. 138.6.
32. 34.2.
33. 18.72.
34. 132.3.
35. 8.82.
36. 96.6.
37. 210.
38. 1530.
39. 35 %.
40. 48 %.
41. 9 %.
42. 9.5 %.
43. 73.5 %.
44. 86.4 %.
45. 2.5 %.
46. 25.8 %.
47. 135 %.
48. 248 %.
49. 290 %.
50. 320 %.
51. 1.5 %.
52. .8 %.
53. .85 %.
54. 12.25 %.
55. 56.47 %.
56. 64.19 %.
57. 44.24 %.
58. 118.18 %.
59. 207.1 %.
60. 79.72 %.
61. 200.89 %.
62. 187.5 %.
63. 165.52 %.

Pages 115–117

1. \$1.54; \$33.20.
2. 152 members.
3. \$305.
4. Smaller.
5. 14; 22.
6. 50 %; $37\frac{7}{9}$ %.
7. From $1\frac{1}{2}$ times to twice as much.
8. 144 %.
9. 309.66 %.
10. 272.94 %.
11. Exports 54.68 %; 47.86 %; Imports 28.72 %; 19.98 %.
12. Cheese, *a.* '17–'18–40 %; *b.* '14–'17–76.56 %; Currants, *c.* 1918–80 %; *d.* 1916–21.88 %; Dates, *e.* '18–82.35 %.

Pages 118–119

3. \$9.50.
4. \$8.
5. \$7.50.
6. $54\frac{4}{9}$ bu.
7. $46\frac{2}{17}$ bu.
8. $41\frac{5}{19}$ bu.

9. 1100 lb.; $1508\frac{2}{14}$ lb.; $1035\frac{5}{7}$ lb.
10. Larger 43.6 ¢ less.
11. 240 lb.
12. 240 lb. dressed.
13. 120 qt.
14. $30.
15. $16.50.

Page 120

1. 96.
2. 96.
3. $7.80.
4. 7.80.
5. 22.50.
6. $50.
7. $8.40.
8. $12.50.
9. 2 lb.
10. 350 lb.

Pages 120–123

1. Food $875; Rent $700; Clothing $770; O. Expenses $525; S. C. and R. $630.
2. Food $525; Rent $300; Clothing $270; O. Expenses $225; S. C. and R. $180.
3. $131\frac{3}{7}$ %.
4. 150.6 % more.
5. $61.35; $163.
6. Wheat 113.73 %; Corn 147.61 %; Barley 93.09 %; Rye 113.55 %; Potatoes 79.04 %; Cotton 92.86 %.
7. 12.09 %.
8. 40.80 %.
9. 2.25 times its former production.
10. 3.25 times.
11. $1,631,250.
12. $2,356,250.
13. 423,400,000 bu.
14. 186,200,000 bu.
15. $287,516,666\frac{2}{3}$ lbs.
16. 62.44 %.
17. 2.79 %.
18. 4,205,737.7 A.
19. Multiply by 1.84; Divide by 1.84.
20. $37.50; $14\frac{2}{3}$ %.
21. 30 ¢.
22. Dressed $\frac{5}{7}$ ¢.
23. $26.47 %.

Pages 124–126

1. $4.25.
2. $16.26.
3. $3.85.
5. $69.44.

Pages 127–128

1. $20.10; $24.50; $29.65.
2. $4.30; $4.30. $5.95; $7.75.

Pages 128–129

1. $79.75.
2. $32.45.
3. $36.50.
4. *a.* Food $8.98; Clothing $11.10; O. Expenses $7.90; Higher Life $3.26; Health $2.50;
   b. Mon. $9.78; Tues. $2.98; Wed. $6.88; Thurs. $.74 ¢; Fri. $.53; Sat. $11.76; Sun. $1.07;
   c. $33.74.

Pages 130–134

17. $32.10.
18. $295.
19. $ .60.
20. $2.85.
21. $165.20.
22. $117.70.

Pages 134–135

1. $28.
2. $17.50.
3. 10 %.
4. 12 %.
5. $16\frac{2}{3}$ %.
6. $22.50.
7. $2125.
8. $16\frac{2}{3}$ %.
9. $315; $120; $148.75.
10. $14\frac{2}{7}$ %; 20 %; 34 %.
12. Divide by .75.
13. $234.60.
14. $94.60.

Page 136

1. $2.63.
2. 14 %.
3. $26.95.
4. $34.24.
5. $48.90.
6. $73.53.
7. $38.56.
8. $87.12.
9. $99. 75.
10. $50.58.
11. $57.92.
12. $55 – ($54.995).
13. 20 %.
14. 10 %.
15. 20 %.

47. 4.89
48. 6.24

Page

By First

1. 67.99.
2. 76.03.
3. 93.52.
4. 43.87.
5. 38.44.
6. 58.84.
7. 30.62.
8. 26.86.
9. 25.16.
10. 28.56.
11. 27.15.
12. 31.17.
13. [illegible]
14. [illegible]

15. [illegible]
16. .0[illegible]
17. .0[illegible]
18. .0[illegible]
19. .09[illegible]

Page 160

1. Alternating \$2.12 and \$2.13.
2. \$10.62–10.63; \$212.50; \$1062.50.
3. \$225; \$450.
4. 20 bonds; 42.
5. \$458,276,650.

Municipal Bonds

1. \$75 per year.
2. \$4500.
3. \$45,000.

Page 162

1. 6.32 %.
2. 4.51 %.
3. 4.796 %.
4. 4.657 %.
5. 6.185 %.

Pages 163–164

2. \$4900.
3. \$7160; \$360.
4. \$5887.50.
5. \$4893.75.
6. \$5317.50.
7. \$6720.
8. \$9146.25.
9. \$6296.88.
10. \$4893.75.
11. \$7565.63.
12. \$7596.88.
13. \$8324.38.
15. \$5056.25.

Page 165

1. Below par.
2. More than par.
3. No. Earns 4.90 %.
4. Yes. Earns 5.49 %.

Page 166

1. \$1; \$51; \$1.02.
2. \$597.55.
3. \$994.90.
4. \$1440.72.
5. \$596.94.
6. \$370.05.
7. \$675.34.
8. \$1391.16.
9. \$2115.90.
10. \$451.53.
11. \$1110.15.
12. \$1402.20.

Pages 169–170

1. \$563.30; \$1248.60; \$2082.50; \$3096.90.
2. \$4994.40.
3. \$8780.70.
4. \$777.52.
5. \$2967.18.
6. \$12,373.11.
7. \$8206.26.
8. \$10,257.83.
9. \$17,803.05.
10. \$82,487.40.
11. \$82,062.60.

Pages 170–171

1. 30 shares; \$15.
2. \$12.50; \$27.50.
4. \$42.50.
5. Between 6 and 7 years.

Pages 171–172

1. \$85 (\$84.88).
2. Rent \$91.50 more.
3. House \$190 more.

Page 173

1. 5000 shares; $\frac{1}{50}$.
2. $\frac{1}{10}$; \$800.
3. 2000 shares; $\frac{1}{2000}$.
4. 15 %.
5. \$160.
6. \$100.
7. \$60,000; \$90.
8. 12 %.

Page 175

1. \$2145; \$2147.50.
2. \$3500.
3. \$185.
4. 7.45 %.
5. $6\frac{2}{3}$ %.

Page 176

1. \$280.
2. \$250.
3. 6.53 %.
4. 6.74 %.
5. \$14,000; 6%; preferred.
6. Preferred 6% better.

Pages 178–180

6. 24%.
7. 1.62 %.
8. \$57.60.
9. \$60.
10. Cottage \$51; Home \$18.
11. \$1600; \$2400.
12. \$23.04; \$9600; \$6000; \$500.
13. $\frac{3}{5}$.
14. \$9600.
15. \$8000.
16. $\frac{3}{4}$.
17. 52.87%; 37.61 %.
18. .42 %.

Pages 181–182

4. \$100.70; \$154.70.
5. \$131.75; \$181.10; \$249.25.
6. \$5141.25. \$7395.75. \$10,986.75.
7. \$5122.16.
8. \$2923.58 more.
9. \$7146.95 more.
10. Insurance \$12,746.62 less.
11. Insurance \$2,768.85 more.

Pages 183–184

1. \$2353.30.
2. \$66.73 less.
3. \$97.82 less.

4. S. V. \$576 less.
5. Ins. \$835.64 less.

Pages 186–188

1. 15 miles; \$1.50; $1\frac{1}{2}\%$.
2. \$48.
3. \$80.75.
4. \$341.67.
5. \$8100.
6. \$96.20.
7. 1.1%.
8. \$.75.
9. \$1.75.
10. \$1.47.
11. 85.73¢.
12. 51.02¢.
13. 51.96¢.
14. 49.63¢.
15. \$1.039.
16. \$56.25.
17. \$92.63.
18. \$173.25.
19. \$148.50.
20. \$83.06.
21. \$198.
22. \$310.63.
23. \$330.27.

Pages 188–189

1. \$2.55.
2. \$4132.
3. \$4500.
4. \$1575.
5. \$10,650.
6. \$10,490,-973.60.
7. \$18,740,846.
8. \$121,012,-301.36.

Page 190

1. \$182,188,-527.
2. \$29,560,-732.68.
3. \$43,185,-703.18.
4. \$54,240,-670.64.

Pages 190–191

1. 9.97%.
2. 34.87%.
3. 25.67 mi.
8. 16.39%.
5. \$2,112,-115.60.
6. \$12,960,043.
7. \$47.12.
8. 18.73%.

Page 194

1. \$370; \$250.
2. \$950; \$734.
3. \$482.

Page 195

1. 96.
2. 96.
3. 125.664.
4. 1809.5616.
5. 3600.
6. 480.
7. 300.
8. 1696.464.
9. 804.2496.
10. 523.6.
11. 88.
13. 297.8321.
14. 2.23A.
15. 70.
16. 484.
17. $7\frac{1}{2}$.
18. 300.

Page 197

1. 4.
2. 5.
3. 5.
4. 8.
5. 17.
6. 5.
7. 6.
8. 4.
9. 3.
10. 5.
11. 6.
12. 10.
13. 18.
14. 12.
15. 20.
17. 4.
18. 8.
19. 6.
20. 5.
21. 10.
22. 10.
23. 12.
24. 16.
25. 4.
26. 10.
27. 25.
28. 15.

Pages 198–199

2. 12 rd.
3. $11\frac{1}{3}$ in.
4. 24 sq. in.
5. 5 ft.
6. 6 ft.
7. 490.
8. 480.
9. 48 pigeons.
10. \$24.
11. \$200.
12. 14 girls; 21 boys.
13. 12 girls; 24 boys.
14. Ralph 60; Donald 30.
15. 30 ft.; 90 ft.
16. Robert 10; James 24.
17. Ralph 64; sister 32.
18. 1.5 qt. cream; 3 qt. milk.
19. James 40; Frank 80; Ralph 120.
20. John 35; Ralph 30.
21. Donald \$30; Ralph \$45.

Pages 199–200

1. 6 in. by 8 in.; 40 ft.
2. $81\frac{1}{4}$ ft.; $91\frac{1}{4}$ ft.
3. About 585 ft.
4. About 540 ft.; 610 ft.
5. 138.4 ft.
6. About 39°.
7. $16,666\frac{2}{3}$ ft.
8. 740.74 ft.

Page 201

2. 150 ft.
3. 35 ft.
4. $22\frac{1}{2}$ ft.
5. 1,500 ft.

Pages 202–203

1. 119 ft.
2. 32.1 ft.; 481.5 sq. ft.
3. 50.4 ft.
4. 3460 ft.
5. 37°.
6. 50°; 40°.
7. 17,021.276 ft.
8. $571\frac{3}{7}$ ft.
9. 77.35 ft.

Pages 204–205

1. $ 33.99.
2. $ 126.88.
3. $ 38; L. E. Barnes; $ 988.

Pages 206–207

4. $ 112.50.
6. $ 31.87–$ 31.88.
7. $ 80.
8. $ 1150; $ 1151.25.
9. $ 70; $92\frac{1}{2}$ ¢ more.
10. $ 75 less.
11. 8 shares; $ 64.
12. Stock $ 4 more; Bond and mortgage.
13. Stock earns more but mortgage safer.
14. $ 28,450.80.
15. $ 1707.048.
16. $ 1610.39.

Pages 208–211

Exercise 1

a. 760.815.
b. 204.89.
c. 2885.792.
d. 34.63.

Exercise 2

a. 863.055.
b. 310.28.
c. 587.6118.
d. 576.4.

Exercise 3

a. 697.294.
b. 523.26.
c. 53,365.38.
d. 63.48.

Exercise 4

a. 833.872.
b. 213.518.
c. 3116.988.
d. 527.9.

Exercise 5

a. 988.996.
b. 282.46.
c. 26,031.92.
d. 463.2.

Exercise 6

a. 850.31.
b. 503.905.
c. 70,840.
d. 573.6.

Exercise 7

a. 685.916.
b. 48.666.
c. 26,482.104.
d. 93.48.

Exercise 8

a. 972.5.
b. 232.107.
c. 5404.616.
d. 798.6.

Exercise 9

a. 493.263.
b. 382.53.
c. 7194.996.
d. 529.7.

Exercise 10

a. 540.974.
b. 563.84.
c. 4093.146.
d. 629.8.

Exercise 11

a. $170\frac{5}{8}$.
b. $146\frac{7}{16}$.
c. 16,965.
d. $769\frac{5}{21}$.

Exercise 12

a. $152\frac{8}{9}$.
b. $998\frac{13}{15}$.
c. $15,362\frac{4}{5}$.
d. $754\frac{31}{32}$.

Exercise 13

a. $199\frac{7}{12}$.
b. $2115\frac{17}{24}$.
c. 29,040.
d. $288\frac{7}{15}$.

Exercise 14

a. $199\frac{23}{24}$.
b. $1087\frac{11}{18}$.
c. $18,043\frac{7}{8}$.
d. $328\frac{37}{40}$.

Exercise 15

a. $208\frac{7}{24}$.
b. $755\frac{17}{24}$.
c. $36,907\frac{1}{5}$.
d. $548\frac{20}{21}$.

Exercise 16

a. $241\frac{11}{24}$.
b. $1189\frac{13}{16}$.
c. $21,811\frac{1}{5}$.
d. $352\frac{29}{40}$.

Exercise 17

a. $193\frac{5}{6}$.
b. $1165\frac{5}{24}$.
c. 32,524.
d. $257\frac{17}{18}$.

Exercise 18

a. $234\frac{5}{24}$.
b. $891\frac{23}{24}$.
c. $16,979\frac{5}{8}$.
d. $380\frac{5}{6}$.

Exercise 19

a. $168\frac{19}{24}$.
b. $1531\frac{7}{10}$.
c. $22,406\frac{3}{7}$.
d. $185\frac{5}{21}$.

Exercise 20

a. $167\frac{4}{5}$.
b. $1092\frac{17}{24}$.
c. 42,008.
d. $510\frac{27}{28}$.